

UNIT - I ①  
ECONOMIC OPERATION

INTRODUCTION :-  
★ ★

Now-a-days all of us depend on electricity and it has become a part & parcel of our life. But the demand & supply are not matching. The design of a power plant should incorporate two important aspects.

- a) The selection & placing of the necessary power generating equipment should be such as to result a maximum return from a minimum expenditure over the working life of plant.
- b) The operation of the plant should be such as to provide cheap, reliable & continuous service.

Old methods :-

- a) "the base load method" where the most efficient unit is loaded to its maximum capability then the second most efficient unit is loaded.
- b) "Best point loading", where units are successively loaded to their lowest heat rate point beginning with the most efficient unit & working down to the least efficient unit etc.

It was recognized in early nineteenth century (2) that the incremental method later known as "the equal incremental method" yielded the most economic results.

\* The main economic factor in the power system operation is the cost of generating real power.

In any power system, this cost has got two components viz.,

- i) Fixed cost
- ii) Variable cost.

i) Fixed cost :- Fixed cost being determined by the capital investment, interest charged on the money borrowed, tax paid, labour charge, salary given to staff & any other expenses that continue irrespective of the load on the power system, and.

ii) The Variable cost :- It is a function of loading on generating units, losses, daily load requirement and purchase or sale of power.

At present, the economic operation of a power system is concerned about minimizing the variable cost factors only as the persons responsible for the operation of a power system have little control over the fixed costs.

A power system is a mix of different modes of generation out of which thermal, hydro & nuclear contribute a major share.



However, economic operation has conventionally <sup>(3)</sup> been considered by proper scheduling of thermal or hydro generation only or of both, as, for the safety of the nuclear stations, these types of stations are required to be operated at a fixed load only & there is little scope to schedule the generation of nuclear type in practice.

### Optimal operation of thermal power units:-

#### \* INPUT-OUTPUT OPERATIONAL CHARACTERISTICS OF THERMAL POWER PLANTS:-

According to the physical principle that with increase in difference between the temperature & pressure of the input & output of any heat-operated device (a turbine), more mechanical power will be developed for the same amount of heat energy ~~input~~ <sup>output</sup> input.

The overall efficiency of thermal units is then determined by measuring the heat input i.e., the electrical energy output.

Conventionally, this represents input-output curves and can be developed for each generating unit

involved shown in fig (i):

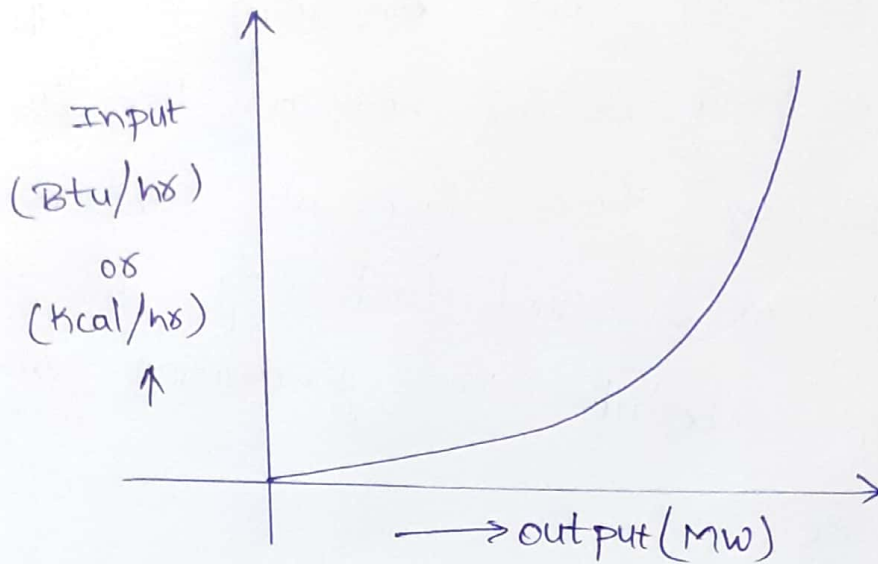


fig (i):- Input - output curve of a thermal-generating unit.

An efficient unit develops a given amount of power with lesser fuel input. Hence, it has become the usual practice to load the more efficient unit before loading the lesser efficient unit.

HEAT RATE CURVE :-

Heat rate curve  $(H_i) P_{Gi}$

is the heat energy in (MKcal) needed to generate one unit of electrical energy.

Heat rate  $(H_i)$   
 $(P_{Gi})$   
MKcal/MWh  
↑

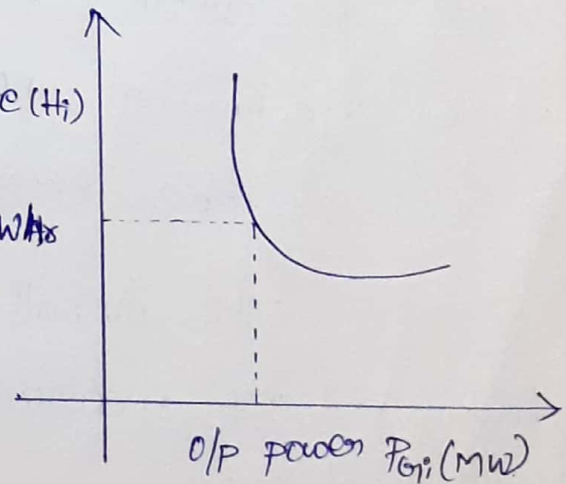


Fig (ii):- Shows the appropriate shape of heat rate curve, which can be obtained experimentally.

fig (ii):- Heat rate curve.



The generating unit efficiency can be defined as (5)

"the ratio of electric energy output generated to fuel energy input". Thus, the generating unit is most efficient at the minimum heat rate, which corresponds to a particular  $P_{Gi}$ . And the curve indicates the increase in the heat rate at low and high power limits.

Incremental Efficiency :-

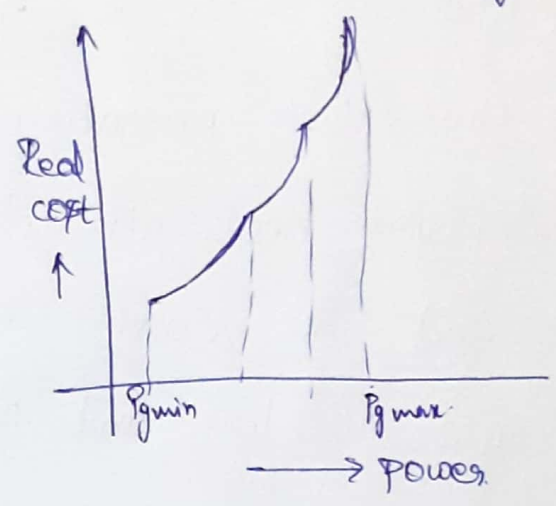
The reciprocal of the incremental fuel rate or heat rate, which is defined as the ratio of output energy to input energy, gives a measure of fuel efficiency for the input.

$$\text{i.e., Incremental efficiency} = \frac{\text{output}}{\text{input}} = \frac{dP_G}{dc}$$

COST CURVE :-

The cost curve can be determined experimentally.

Fig(i) show the fuel cost curve, the discontinuity occurs when the output has to be excited by using additional boilers, steam condensers, discontinuities also appears, if the cost represents the operation of an entire power station, so that cost has discontinuities on paralleling of generators.

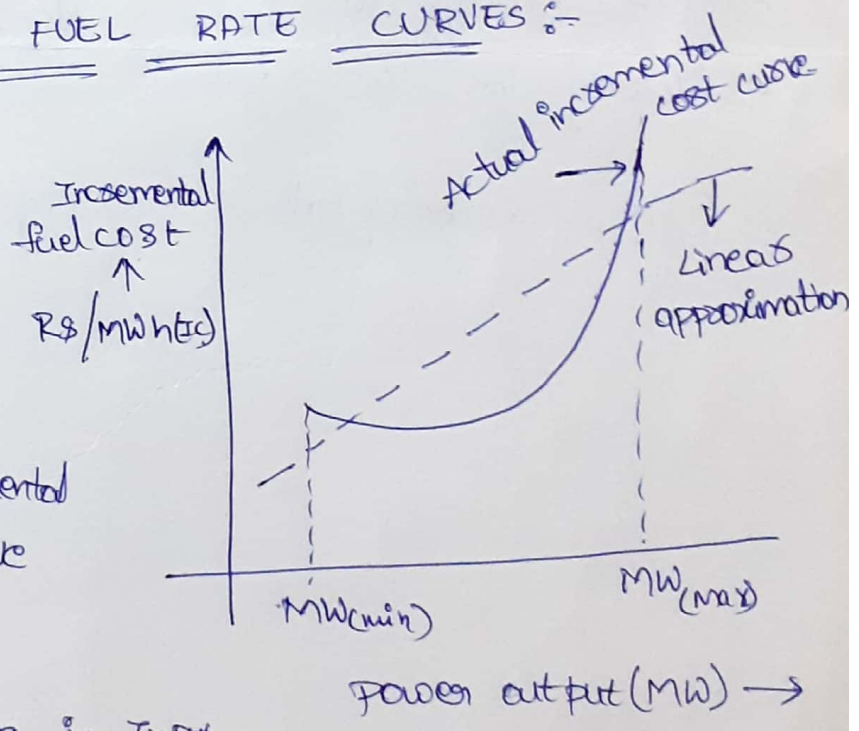


Fig(ii): cost curve

INCREMENTAL FUEL COST (os)

INCREMENTAL FUEL RATE CURVES :-

The input - output curves being obtained from the operating data of the power station can be utilized to get the "incremental fuel rate" (IFR or IR) curve from the relation



$$IFR = \frac{\text{Incremental change in Input}}{\text{Incremental change in output}}$$

Fig(iii):

Thus by calculating the shape of the input-output curves at various points of operation. The profile of IFR can be obtained. The input-output curve of a unit



(it consists of boiler, turbine, generator) can be expressed in million kilo calories/hour or directly in terms of rupees per hour versus output in MW. The cost curve can be determined experimentally. A typical curve is shown in fig (iii) where MW(min) is the minimum loading limit below which it is uneconomical to operate the unit & MW(max) is the maximum output limit.

An analytical expression for operating cost can be written as  $C_i(P_{Gi})$  ₹/hour at output  $P_{Gi}$

where the suffix "i" stands for the number. It generally suffices to fit a second-degree polynomial i.e.

$$C_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad \text{₹/hr} \quad \longrightarrow \text{eq (1)}$$

the slope of the cost curve i.e.  $\frac{dC_i}{dP_{Gi}}$  is called incremental fuel cost (IC) and is expressed in units of ₹/MWH (₹/MWhrs).

A typical plot of incremental fuel cost versus output power shown in fig (iii). If the cost is approximated as a quadratic as eq (1), we have

$$(IC)_i = b_i + 2c_i P_{Gi} \quad \longrightarrow \text{eq (2)}$$

A linear relationship for better accuracy incremental fuel cost may be expressed by a number of short line segments. Alternatively we can fit a polynomial of suitable degree to represent the IC curve in the

inverse for as

(8)

$$P_{Gi} = \alpha_i + \beta_i (IC)_i + \gamma_i (IC)_i + \dots \quad \text{MW}$$

→ eq (3)

### INCREMENTAL PRODUCTION COST :-

Incremental production cost consists of the incremental fuel cost plus the incremental cost of labour, supplies, maintenance and water.

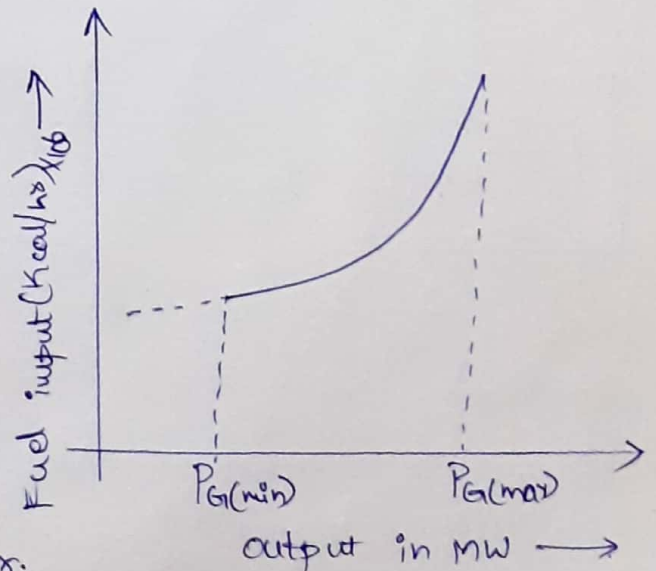
It is difficult to express exactly these costs as a function of output & also they form generally a small fraction of the incremental cost of fuel, the incremental cost of production will rather to be considered equal to the incremental cost of fuel.

The incremental efficiency is defined as the reciprocal of incremental fuel rate (or) incremental production cost and is given as incremental efficiency =  $\frac{\text{output}}{\text{input}} = \frac{dP}{dF}$ .

### INPUT - OUTPUT CHARACTERISTICS :-

The idealized form of input-output characteristics of a steam unit is shown in fig (i) :-

It establishes the relationship between the energy input to the turbine and the energy output from the electrical generator.





The input to the turbine shown on the ordinate may be either in terms of the heat energy requirement, which is generally measured in Btu/hr or kcal/hr or in terms of the total cost of fuel per hour in Rs/hr. The output is normally the net electrical power output of that steam unit in kW or MW.

In practice, the curve may not be very smooth, and from practical data, such an idealized curve may be interpolated. The steam turbine-generating unit curve consists of minimum and maximum limits in operation, which depend up on the steam cycle used, thermal characteristics of material, the operating temperature, e-t-c.

Units of turbine input :-

In terms of heat, the unit is  $10^6$  kcal/hr (or) Btu/hr or in terms of the amount of fuel, the unit is tons of fuel/hr, which becomes millions of kcal/hr.

COST CURVES :-

To convert the input-output curves into cost curves, the fuel input per hour is multiplied with the cost of the fuel. (expressed in Rs/million kcal)

$$\begin{aligned}
 \text{i.e. } & \frac{\text{kcal} \times 10^6}{\text{hr}} \times \text{Rs / million kcal} \\
 & = \text{million kcal/hr} \times \text{Rs. / million kcal} \\
 & = \text{Rs. / hr.}
 \end{aligned}$$

# INCREMENTAL FUEL COST CURVE :-

From the input-output curves, the incremental fuel cost (IFC) curve can be obtained.

The IFC is defined as the ratio of a small change in the input to the corresponding small change in the output.

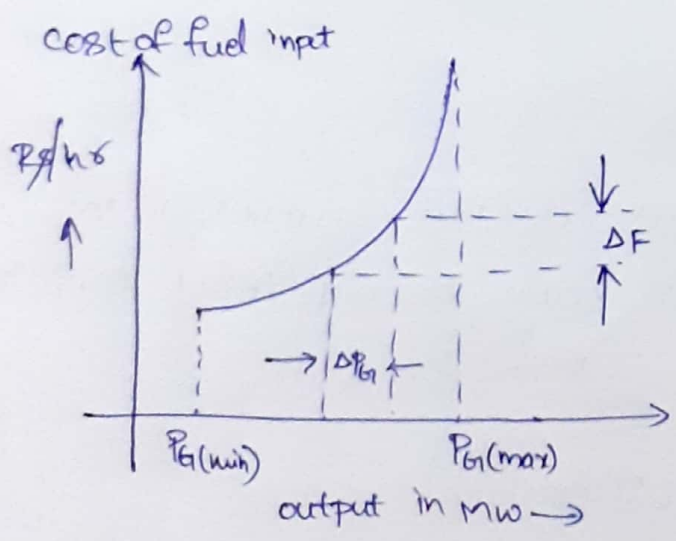
$$\text{Incremental fuel cost} = \frac{\Delta \text{ input}}{\Delta \text{ output}} = \frac{\Delta F}{\Delta P_G}$$

where  $\Delta \rightarrow$  small changes

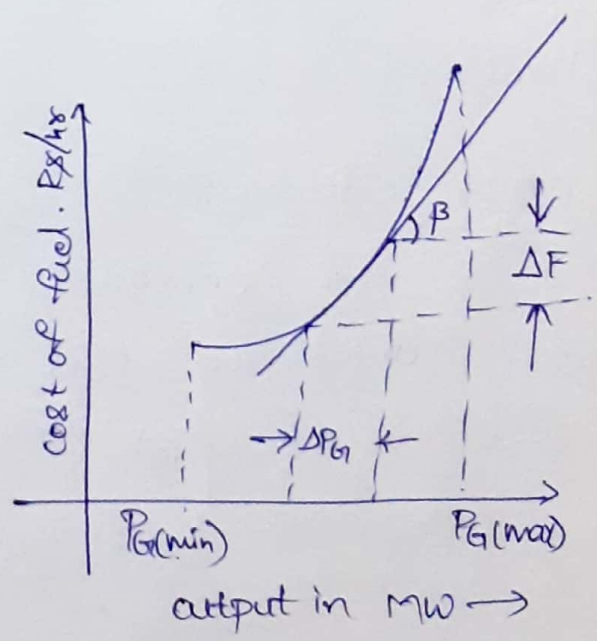
As the ' $\Delta$ ' quantities become progressively smaller, it is

seen that  $\text{IFC is } \frac{d(\text{input})}{d(\text{output})}$  and is expressed in  $\text{Rs/MWh}$ .

A typical plot of the IFC versus output power is shown in fig(a):



(a)



(b)



fig:- (a) incremental cost curve

(b) incremental fuel cost characteristic in terms of the slope of the input -

The IFC is now obtained as

$(IC)_i$  = slope of the fuel cost curve

i.e.,  $\tan \beta = \frac{\Delta F}{\Delta P_i}$  in Rs/mwh

The IFC (IC) of the  $i$ th thermal unit is defined, for a given power output, as the limit of the ratio of the increased cost of fuel or input (Rs/hr) to the corresponding increase in power output (mw), as the increasing power output approaches zero.

i.e.  $(IC)_i = P_{Gi} \lim_{\Delta P_{Gi} \rightarrow 0} \frac{\Delta F_i}{\Delta P_{Gi}} = \frac{dF_i}{dP_{Gi}}$  (Rs)

$(IC)_i = \frac{dc_i}{dP_{Gi}} \left[ \frac{dF_i}{dP_{Gi}} = \frac{dc_i}{dP_{Gi}} = \text{Incremental fuel cost of the } i\text{th unit} \right]$

where,  $c_i$  = cost of fuel of the  $i$ th unit

$P_{Gi}$  = Power generation output of that  $i$ th unit.

Mathematically, the IFC curve expression can be obtained from the expression of the cost curve.

cost-curve expression,

$c_i = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i$  (second-degree polynomial)

The IFC,  $\frac{dc_i}{dP_{Gi}} = (IC)_i = a_i P_{Gi} + b_i$  (linear approximation) for all  $i = 1, 2, 3, \dots, n$ .

where  $\frac{dc_i}{dP_{Gi}}$  is the ratio of incremental fuel energy input in Btu to the incremental energy output in kwh, which is called "the incremental heat rate".

The fuel cost is the major component and the remaining costs such as maintenance, salaries etc. will be of very small percentage of fuel cost; hence, the IFC is very significant in the economic loading of a generating unit.

SYSTEM CONSTRAINTS :- (In Economic OPERATION OF POWER SYSTEM)

The following are the various constraints before taking up the economic dispatching problem

~~there are two types of constraints~~

- 1) ~~Equality constraints~~ (~~or~~) ~~Primary constraints~~
- 2) ~~Inequality constraints.~~

1) Equality constraints :-

- 1) Primary constraints (~~or~~) Equality
- 2) secondary constraints (~~or~~) Inequality.
- 3) Dynamic constraints.
- 4) Space capacity constraints
- 5) operational constraints.



1) Primary constraints :- (or) Equality constraints :-

These constraints arise out of the necessity for the system to balance the load demand and generation. They are also called "equality constraints".

If  $P_i$  &  $Q_i$  are the scheduled electrical generation,  $P_{loadi}$  and  $Q_{loadi}$  are the respective load demands, it is obvious that the following equations must be satisfied at the load bus.

$$P_i - P_{loadi} - P_L = M_i = 0 \quad \text{--- (1)}$$

$$Q_i - Q_{loadi} - Q_L = N_i = 0 \quad \text{--- (2)}$$

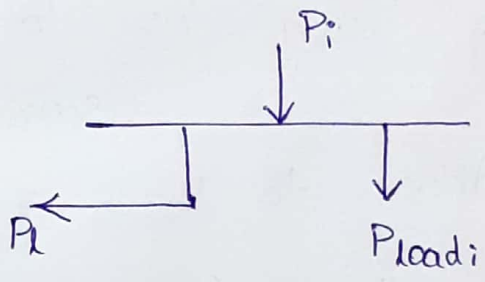


Fig (i) :- Real power position at load bus.

where,  $M_i$  &  $N_i$  represent the power residuals at bus- $i$  and  $P_L$  &  $Q_L$   $\rightarrow$  Power flow to the neighbouring system

$$P_L = \sum_{j=1}^N V_i V_j Y_{ij} \cos(\delta_{ij} - \theta_{ij}) \quad \text{--- eq (3)}$$

$$Q_L = \sum_{j=1}^N V_i V_j Y_{ij} \sin(\delta_{ij} - \theta_{ij}) \quad \text{--- eq (4)}$$

2) Secondary constraints (or) Inequality constraints :-

These constraints arise due to physical & operational limitations of respective units and components and are known as "inequality constraints".

Power inequality constraints are applicable for proper operation; for each generator we should have a minimum & maximum permissible output and the unit production should be constrained to ensure that

$$\begin{array}{l}
 P_{i \min} \leq P_i \leq P_{i \max} \\
 Q_{i \min} \leq Q_i \leq Q_{i \max}
 \end{array}
 \quad
 \begin{array}{l}
 i = 1, 2, \dots, N_p \\
 i = 1, 2, \dots, N_q
 \end{array}$$

$N_p$  &  $N_q$  being the total number of real and reactive sources in the system.

In addition to these inequality constraints, another constraint  $P_i^2 + Q_i^2 \leq (S_{i \text{ rated}})^2$  must be satisfied, where  $S_{i \text{ rated}}$  denotes the complex power capacity of the generating unit without any over loading

3) Dynamic constraints :-

these constraints arise where fast changes in generation are required for picking up the increasing load demand, here,

$$\left| \frac{dP_i(t)}{dt} \right|_{\min} \leq \left| \frac{dP_i(t)}{dt} \right| \leq \left| \frac{dP_i(t)}{dt} \right|_{\max}$$

Similarly for reactive power constraints.

$$\left| \frac{dQ_i(t)}{dt} \right|_{\min} \leq \left| \frac{dQ_i(t)}{dt} \right| \leq \left| \frac{dQ_i(t)}{dt} \right|_{\max}$$



4) spare capacity constraints :-

In order to account for the errors in load prediction, any sudden & fast change in load demands and the inadvertent loss of scheduled generation, spare capacity constraints are frequently utilized.

In this constraint, the total generation available at any time should be in excess of the total anticipated load demand and any system loss by an amount not less than a specified minimum spare capacity 'P<sub>sps</sub>'

$$P_{ig} \geq \sum_{i=1}^N P_L + P_{sps} + P_{load i}$$

For groups of generators, when all plants are not equally operationally suitable for taking up additional load, this constraint is then given by

$$P_{ig} \geq \sum P_L + P_{spg} + P_{load i}$$

where, P<sub>spg</sub> → spare capacity generation for the specified generators (s).

5) thermal constraints of transmission lines :-

these constraints arise when power injection (+S<sub>max</sub>) or power drawal (-S<sub>max</sub>) is allowed such that

$$|S_{min}| \leq |S_{tx}| \leq |S_{max}| ; i = 1, 2, \dots (t_x)_n$$

where (t<sub>x</sub>)<sub>n</sub> represents the number of branches and S<sub>tx</sub> the branch power transfer in MW.

6) Bus voltage & Angle constraints :-

These constraints arise in order to maintain voltage profile at load bus & limiting the overload capacity. Here,

$$\begin{array}{l}
 V_{min} \leq V_i \leq V_{max}, \quad i = 1, 2, \dots, N \\
 \delta_{imin} \leq \delta_i \leq \delta_{imax} \quad j \neq 1, j = 2, 3, \dots, M.
 \end{array}$$

where,

N → the number of units  
M → the number of loads in the system.

7) operational constraint :-

In case the transformer tap position needs to be included for optimization, the tap position "a<sub>i</sub>" should lie within the range available in the transformer

i.e., 
$$a_{imin} \leq a_i \leq a_{imax}$$

OPTIMUM GENERATION ALLOCATION WITH LINE LOSSES NEGLECTED

(OR)

Economic operation of thermal units without considering

Line losses :-

Let N = total number of thermal generating units connected to single bus having net electrical loading P<sub>load</sub>

F<sub>i</sub> = Input of the i<sup>th</sup> unit in terms of cost rate.

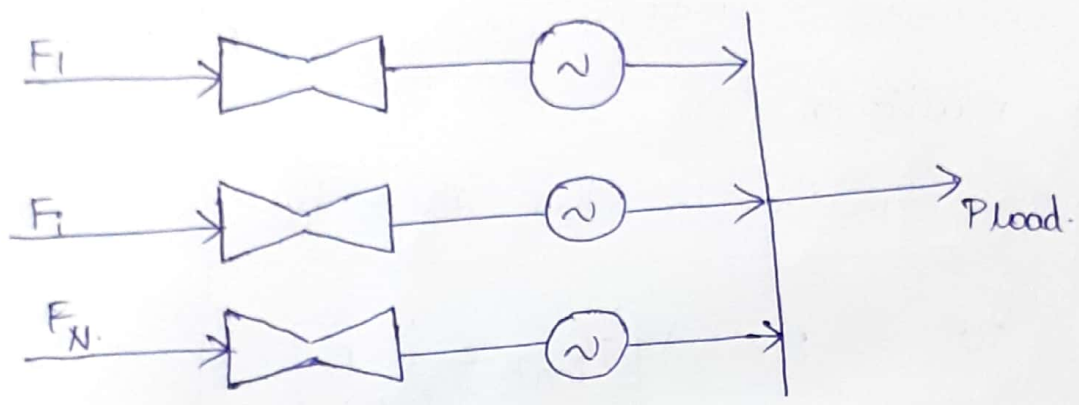
P<sub>i</sub> = <sup>Electrical</sup> output of the i<sup>th</sup> unit.



We know that, the total cost rate is the summation of cost rates of each individual unit

i.e., 
$$F = F_1 + F_2 + \dots + F_i + \dots + F_N = \sum_{i=1}^N F(P_i)$$

→ eq (1)



Fig(a):- Block diagram of the power system comprising thermal generators.

The objective is to determine the criterion of economic loading of thermal units.

constraint :-

sum of output power ( $\sum P_i$ ) = Net load demand (Pload)

Assumption :- All internal losses as well as the transmission loss (or line loss) are neglected.

Economic operation is only possible when an objective function (i.e., the total cost of input (F) for supplying the net load) will be minimised subject to the specified constraint. the constraint optimisation can be done by assuming a constraint function 'phi' such that

$$P_{load} - \sum_{i=1}^N P_i = \phi \quad \longrightarrow \text{eq (2)}$$

In order to achieve the minimum value of the objective function ( $F$ ), use of the "Lagrange function ( $L$ )" is conventional. Here, the cost function (i.e. the objective function  $F$ ) is added to the constraint function ( $\phi$ ) after the constraint function is multiplied by an undetermined multiplier ( $\lambda$ ).

The Lagrangian function is then given by.

$$L = F + \lambda \phi = F + \lambda \left[ P_{\text{load}} - \sum_{i=1}^N P_i \right]$$

→ eq (3)

The necessary condition for the minimum value of the objective function results, when the first derivative of  $L$  with respect to each of the independent variable is zero. i.e.  $F' = 0$

∴ taking the partial derivative of the Lagrangian function with respect to power output values (one at a time) to get  $N$  number of equations for the  $N$ -number of plants operating in the system.

thus, for the  $i$ th plant

$$\frac{\partial L}{\partial P_i} = \frac{dF_i(P_i)}{dP_i} - \lambda = 0$$

$$\Rightarrow \lambda = \frac{dF_i(P_i)}{dP_i} \quad \rightarrow \text{eq (4)}$$



The necessary condition for the existence of a minimum cost operating condition for the thermal based energy generating system is thus obtained in eq(4).

When the incremental cost rate,  $\frac{dF_i(P_i)}{dP_i}$  of all the units is equal to  $\lambda$ .

i.e  $\boxed{\frac{dF_i(P_i)}{dP_i} = \lambda_i}$  { for all units },

the Lagrangian multiplier for 'N' numbers of such units, economic operation is thus attained when

$$\boxed{\frac{dF_1(P_1)}{dP_1} = \frac{dF_2(P_2)}{dP_2} = \dots = \frac{dF_i(P_i)}{dP_i} = \dots = \frac{dF_N(P_N)}{dP_N} = \lambda}$$

→ eq(5)

Computer solution of the economic operation

Problem :-

The simplest procedure for the economic scheduling of thermal power generating plants, with losses neglected, is conventionally the  $\lambda$ -iteration method, the algorithm being represented below.

- 1) Assume a suitable value of  $\lambda^0$  [i.e,  $IC = (IC)^0$ ] this value should be more than the largest intercept of the incremental characteristics of the various generators
- 2) compute the individual generations  $P_{G1}, P_{G2}, \dots, P_{Gi}$

Corresponding the incremental cost of production

from 
$$\frac{dc_1}{dP_{G1}} = \frac{dc_2}{dP_{G2}} = \dots = \frac{dc_n}{dP_{Gn}} = \lambda$$

In case generation at any of the buses is violated, the generation of that generator is fixed at the limit violated during that iteration and the remaining load is distributed among the remaining generators.

3) check if the equality  $\sum_{i=1}^n P_{Gi} = P_D$  is satisfied.

here  $P_{Gi}$  incremental cost rate for a thermal generator.

4) if not make a second guess  $\lambda'$  & repeat the above steps.

The selection of  $\lambda'$  in the step must of course be guided by the result in step 3 for example.

If we find that total generation is less than  $P_D$

then correct the value of  $\lambda'$  to be selected

would be  $\lambda_{corrected} > \lambda'$

If equality is satisfied the generations are calculated in step-2 give the optimum operating strategy.

We know that

$$C_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2$$

$$\frac{dc_i}{dP_{Gi}} = 2c_i P_{Gi} + b_i = \lambda$$

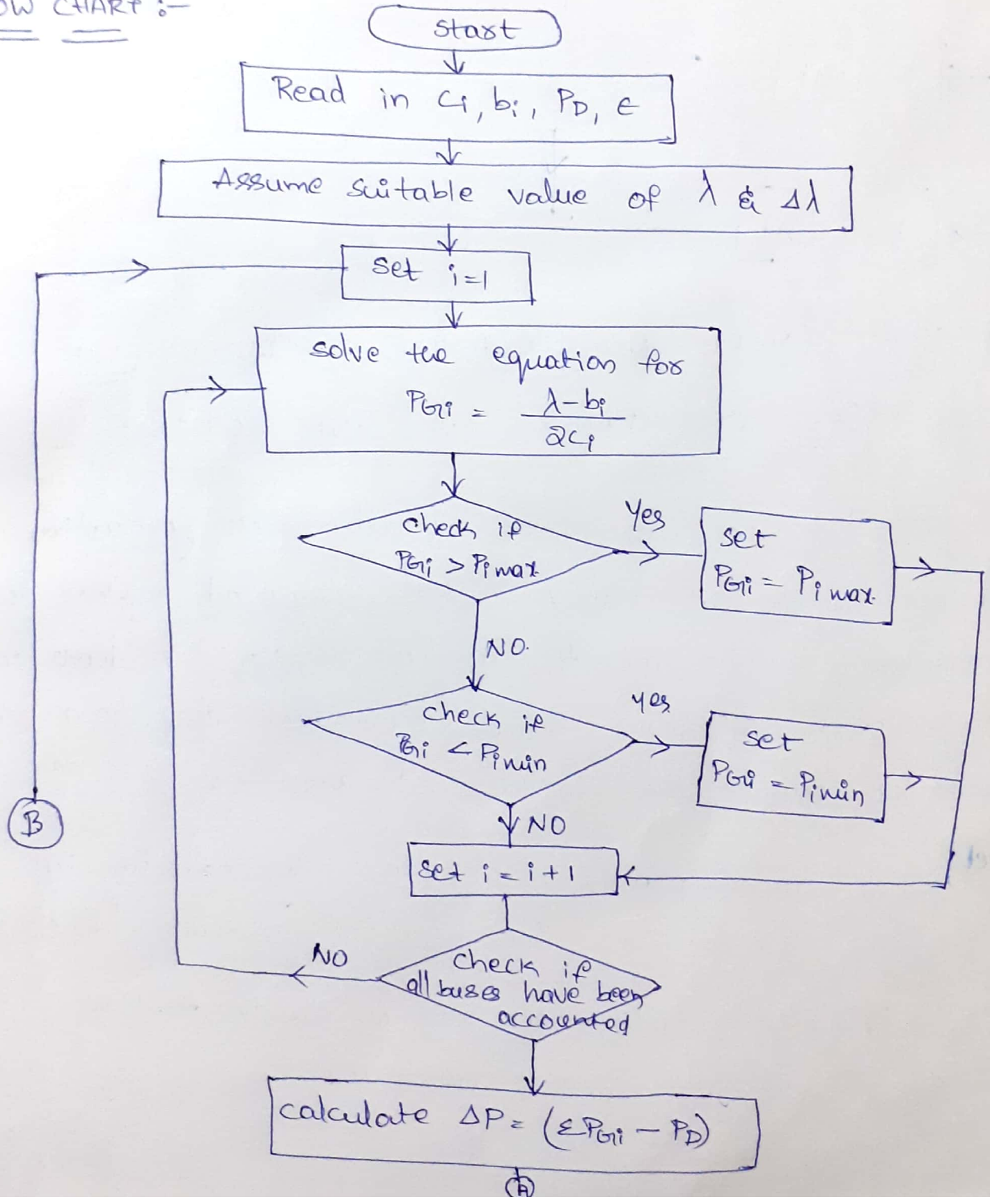
$$\Rightarrow 2c_i P_{Gi} + b_i = \lambda$$

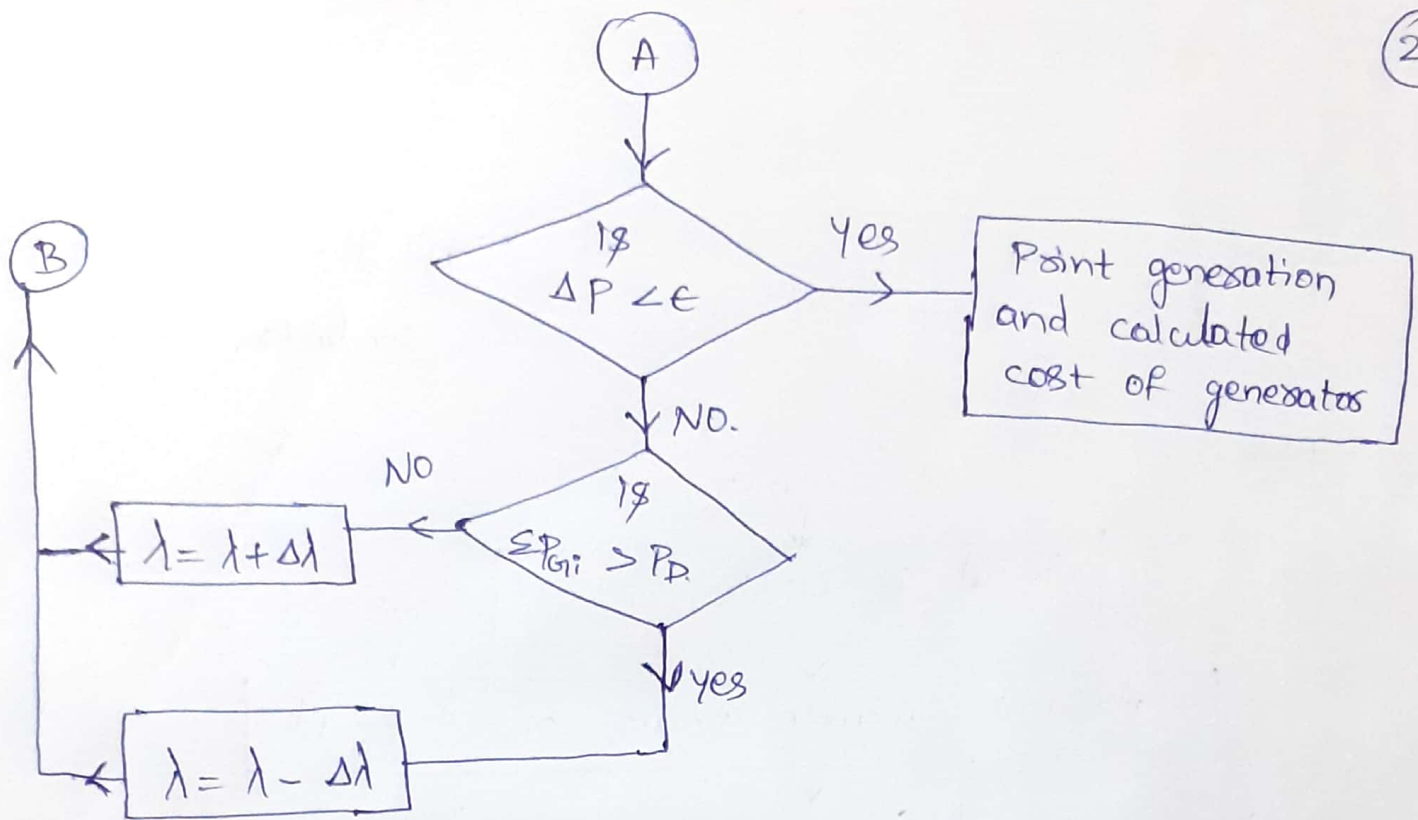


$$\Rightarrow P_{Gi} = \frac{\lambda - b_i}{\alpha c_i}$$

where,  $\alpha c_i$  = slope of incremental production cost curve  
 $b_i$  = Intercept of incremental production cost curve.

FLOW CHART :-





### OPTIMUM GENERATION ALLOCATION INCLUDING THE EFFECT OF TRANSMISSION LINE LOSSES :-

Consider the economic allocation of generation between different plants in an integrated system, the transmission losses are to be considered. This leads to the dispatch of power in an economical way so as to make the overall cost to be minimum.

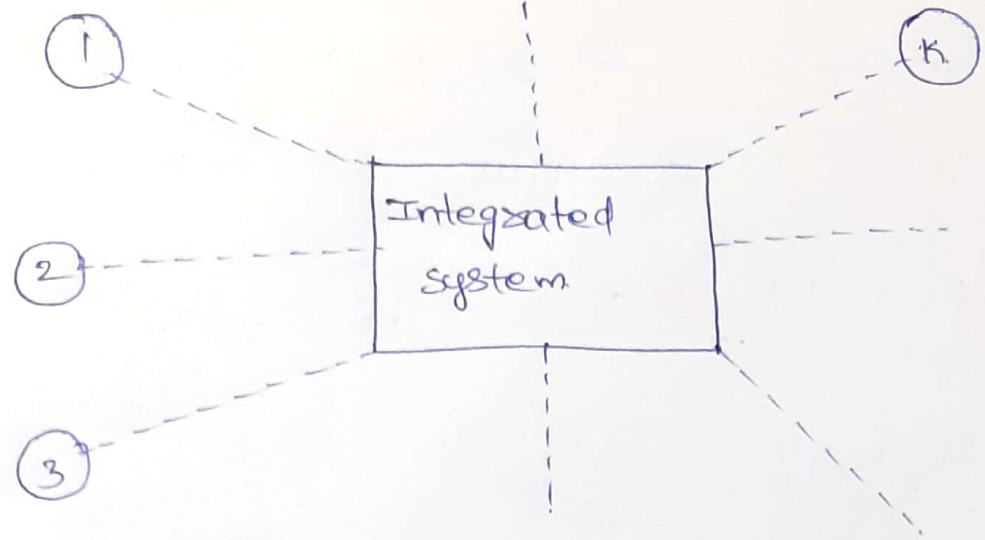
Let,  $N =$  no of plants in a system interconnected by transmission lines & ties. (as shown in fig(c))

$P_1, P_2, \dots, P_N =$  Generation of the  $N$ -plants. in MW.

$P_{load} =$  Total load (constant)

$P_l =$  loss in the lines





Fig(1) :- An integrated power pool.

The constraint equation is given by

$$P_L + P_{load} = \sum_{i=1}^N P_i$$

$$\Rightarrow P_L + P_{load} - \sum_{i=1}^N P_i = \phi_R \longrightarrow \text{eq(1)}$$

where,  $\phi_R$  = Residual power and should approach zero during steady state power system operation.

∴ For steady state operation, eq(1) becomes

$$P_L + P_{load} - \sum_{i=1}^N P_i = 0 \longrightarrow \text{eq(2)}$$

Let,

$F$  = Net fuel input cost/hours

= summation of fuel costs/hr of each of the units governed by the generated power  $P_i$

i.e

$$F = \sum_{i=1}^N F_i(P_i) \text{ Rs/hr} \longrightarrow \text{eq(3)}$$

In order to optimise real power generation, Applying the Lagrangian technique, we get,

$$L = F + \lambda \phi \longrightarrow \text{eq (4)}$$

$\lambda$  = Lagrangian multiplier

w.k.t, the condition for minimum cost function is

$$\frac{\partial L}{\partial P_i} = 0 \longrightarrow \text{eq (5)}$$

$\therefore$  Sub eq (1) in eq (4) we get

$$\Rightarrow \text{Eq (4) becomes, } L = F + \lambda \left[ P_1 + P_{load} - \sum_{i=1}^N P_i \right]$$

$$\Rightarrow \frac{\partial L}{\partial P_i} = \frac{\partial F_i(P_i)}{\partial P_i} + \lambda \left[ \frac{\partial P_1 + 0}{\partial P_i} - 1 \right] = 0$$

$$\Rightarrow \frac{dF_i(P_i)}{dP_i} + \lambda \left[ \frac{\partial P_1}{\partial P_i} - 1 \right] = 0$$

$$\Rightarrow \frac{dF_i(P_i)}{dP_i} - \lambda \left[ 1 - \frac{\partial P_1}{\partial P_i} \right] = 0$$

$$\Rightarrow \frac{dF_i(P_i)}{dP_i} - \lambda + \lambda \frac{\partial P_1}{\partial P_i} = 0.$$

$$\Rightarrow \boxed{\frac{dF_i(P_i)}{dP_i} + \lambda \frac{\partial P_1}{\partial P_i} = \lambda} \longrightarrow \text{eq (6)}$$



eq(6) represents the modified economic operation criterion for the thermal plants with transmission losses considered.

eq(6) may be written as

$$\frac{dF_i(P_i)}{dP_i} - \lambda \left[ 1 - \frac{\partial P_L}{\partial P_i} \right] = 0$$

$$\Rightarrow \frac{dF_i(P_i)}{dP_i} = \lambda \left[ 1 - \frac{\partial P_L}{\partial P_i} \right]$$

$$\Rightarrow \frac{dF_i(P_i)}{dP_i} \left[ \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \right] = \lambda \longrightarrow \text{eq(7)}$$

$$\Rightarrow \left[ \frac{dF_i(P_i)}{dP_i} * (PF_i) = \lambda \right] \longrightarrow \text{eq(7a)}$$

$$\therefore \text{penalty factor, } PF_i = \left[ 1 - \frac{\partial P_L}{\partial P_i} \right]^{-1} \longrightarrow \text{eq(8)}$$

eq(7) (or) eq(7a) is known as exact coordination equation.

In eq(7)  $\frac{dF_i(P_i)}{dP_i}$  = incremental cost

$\frac{\partial P_L}{\partial P_i}$  = incremental transmission loss.

For 'N' number of plants the coordination equations are given as

$$\left. \begin{aligned} \frac{dF_1(P_1)}{dP_1} - \lambda \left[ 1 - \frac{\partial P_L}{\partial P_1} \right] &= 0 \\ \frac{dF_2(P_2)}{dP_2} - \lambda \left[ 1 - \frac{\partial P_L}{\partial P_2} \right] &= 0 \\ \vdots \\ \frac{dF_N(P_N)}{dP_N} - \lambda \left[ 1 - \frac{\partial P_L}{\partial P_N} \right] &= 0 \end{aligned} \right\} \longrightarrow \text{eq (9)}$$

while the constraint equation is given by

$$\sum_{i=1}^N P_i - P_{load} + P_L = \phi_R \longrightarrow \text{eq (10)}$$

If  $\frac{\partial P_L}{\partial P_i}$  is known, then generator output power may be adjusted to satisfy the following equation

$$\frac{dF_i(P_i)}{dP_i} \left[ \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \right] = \lambda; \quad i = 1, 2, \dots, N$$

The optimum economy is thus achieved when the product of the incremental fuel cost times the Penalty factor is the same for all plants,

$$\text{i.e., } \frac{dF_1(P_1)}{dP_1} \cdot PF_1 = \frac{dF_2(P_2)}{dP_2} \cdot PF_2 = \dots = \frac{dF_N(P_N)}{dP_N} \cdot PF_N = \lambda$$

→ eq (11)

here,  $\lambda =$  incremental cost of the received power in unit of (currency/MWh)

→ hence during economic operation of plants with



losses being considered.

$$\lambda = \frac{\text{Incremental fuel cost}}{1 - \text{Incremental transmission loss}}$$

Computer Approach:-

Algorithm to obtain the generation schedule with transmission losses considered is as follows.

Step:1:- set starting values of generation  $P_1, P_2, \dots$  that satisfies the constraint equation.

$$P_{load} + P_L - \sum_{i=1}^n P_i = \phi_R \quad \left\{ \begin{array}{l} \text{for first iteration,} \\ P_L = 0 \end{array} \right.$$

Step:2:- calculate the incremental losses

$$\frac{\partial P_L}{\partial P_1}, \frac{\partial P_L}{\partial P_2}, \dots \text{ as well as total loss}$$

$$P_L = n_1 P_1^2 + n_2 P_2^2 + \dots + n_i P_i^2 + \dots + n_n P_n^2$$

where,  $n_1, n_2, \dots$  are the pure functions representing P-U contribution of the respective generators to the loss.

Step:3:- calculate  $\frac{dF_i(P_i)}{dP_i}$  from the given cost function equation

( $F_i = \alpha P_i^2 + \beta P_i + \gamma$ ) and write the sets of linear equations from,

$$\frac{dF_i(P_i)}{dP_i} = \lambda \left[ 1 - \frac{\partial P_L}{\partial P_i} \right], \left\{ \begin{array}{l} \frac{\partial P_L}{\partial P_i} \text{ being known from} \\ \text{step-2} \end{array} \right.$$

also,  $\sum_{i=1}^n P_i = P_{load} + P_L$  (ideally)  $\left| \sum_{i=1}^n P_i - P_{load} - P_L \right| \leq \epsilon$ , a tolerance

solving these equations  $\lambda$  is obtained along with obtain generators

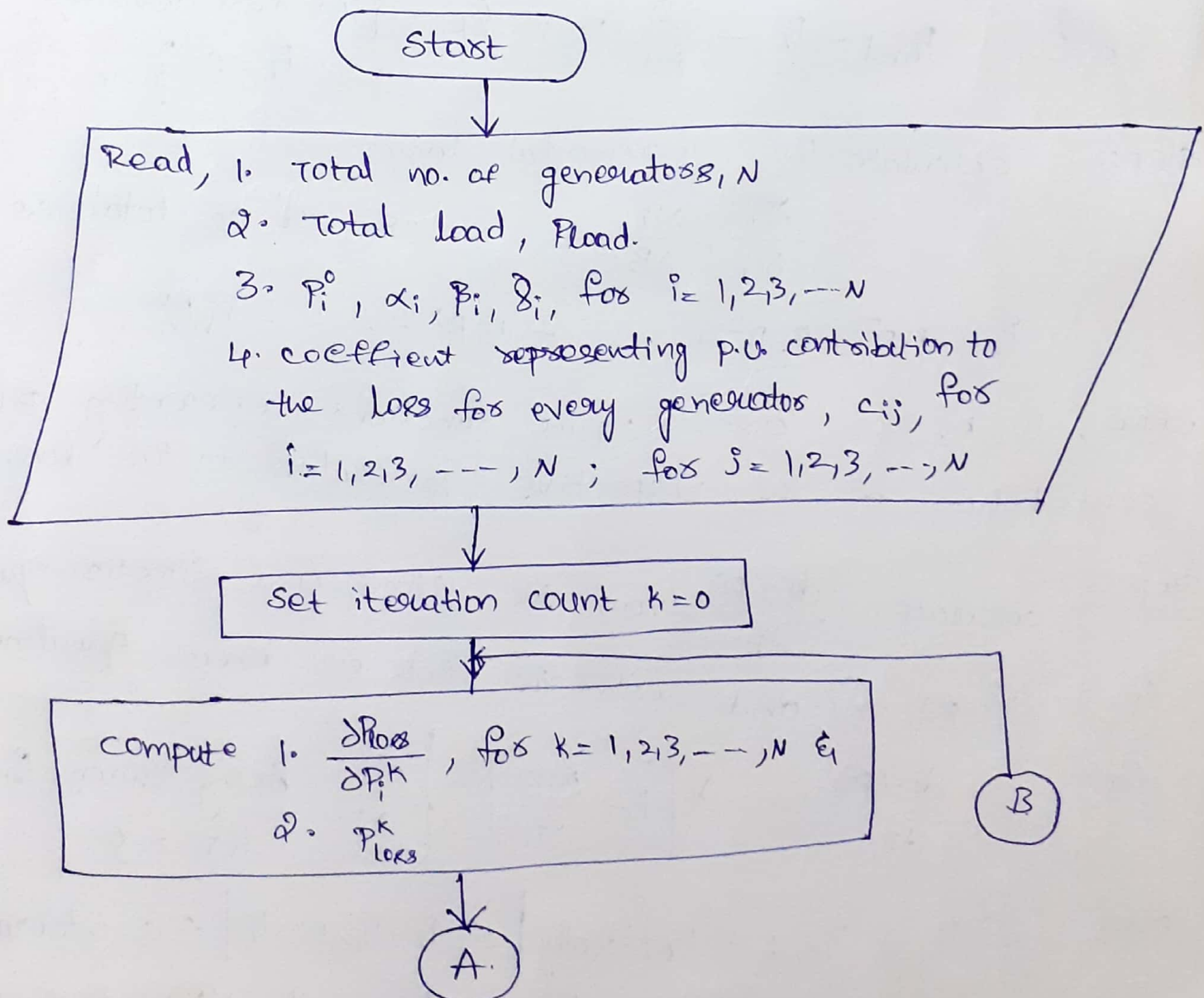
outputs  $P_1, P_2, \dots$

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Step-4:- compare obtained values of  $P_1, P_2, \dots$  to the values used at the start of step-2. If there is a significant change, proceed to step-2 with  $P_1, P_2, \dots$  obtained in step-3. otherwise go to step-5

Step-5:- stop.

The flowchart of the computer program developed to find economic generation schedule considering transmission line losses.





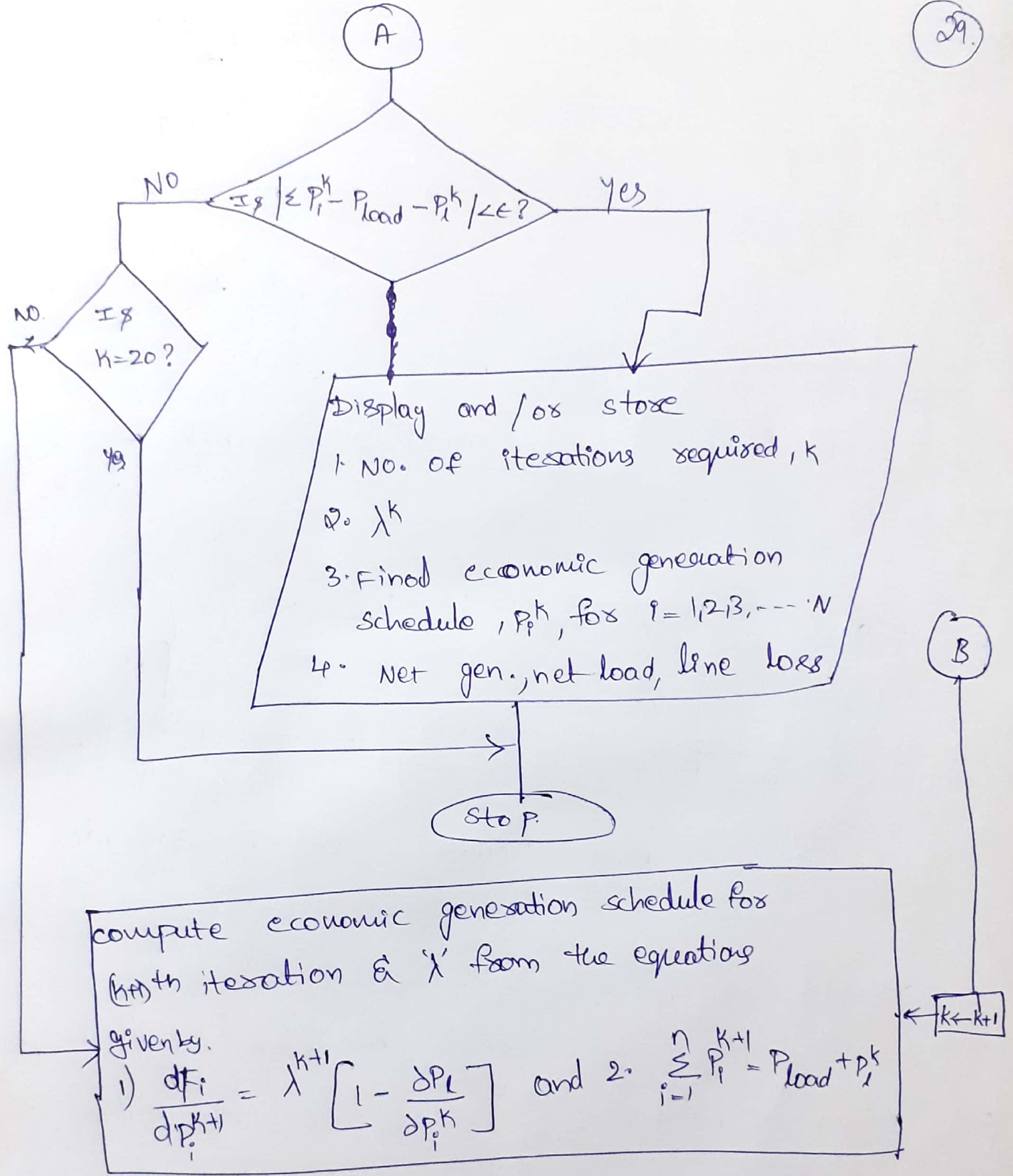


Fig:- Flow chart to find economic generation schedule considering transmission line losses

# GENERAL TRANSMISSION LINE LOSS FORMULA AND

(30)

## LOSS COEFFICIENTS :- (B-coefficients)

consider a power system supplying 'n<sub>l</sub>' loads

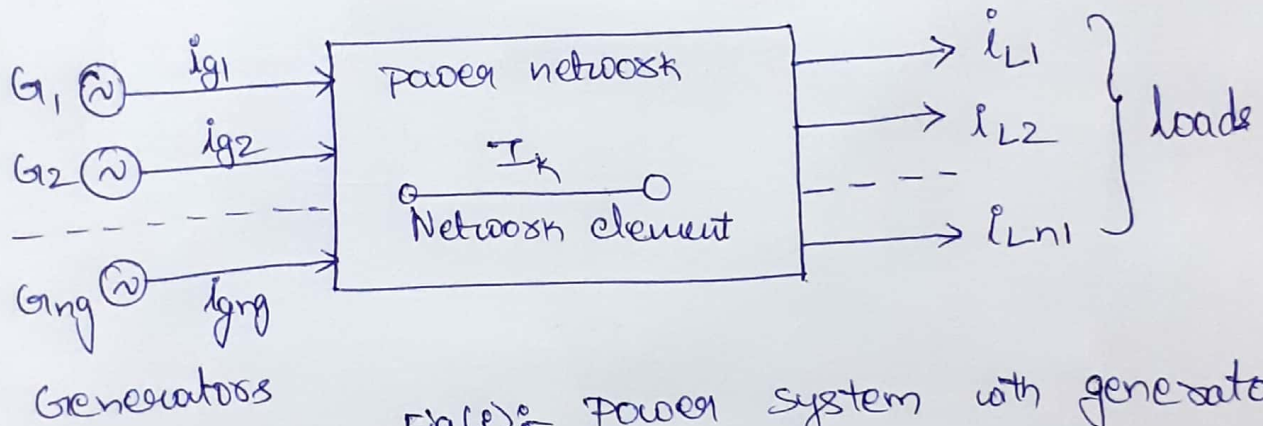
Let

$i_{L1}, i_{L2}, \dots, i_{Ln}$  = load currents supplied by 'n<sub>g</sub>' generators

n<sub>g</sub> = no of generators.

$i_{g1}, i_{g2}, \dots, i_{gn}$  = generator currents

this is shown in fig(i):-



Fig(i):- Power system with generators and load currents.

consider a network element 'k' (an inter connected line in the system) carrying current 'I<sub>k</sub>'.

Let generator-1 (G<sub>1</sub>), alone supply the entire load current I<sub>L</sub>, where,

$$I_L = i_{L1} + i_{L2} + \dots + i_{Ln}$$

$$\Rightarrow I_L = \sum_{j=1}^{n_l} i_{Lj} \quad \longrightarrow \text{eq(1)}$$



under this condition,

Let, the current in k =  $i_{k1}$

In a similar manner if each of the 'ng' generators operating alone also supply the total load current  $I_L$  while the rest of the generators are disconnected the current carried by the network element 'k' changes from  $i_{k1}$  to  $i_{k2}$ ,  $i_{k3}$  to  $i_{kng}$ .

Let the ratio of  $i_{k1}$  to  $I_L$  is  $d_{k1}$

i.e.  $d_{k1} = \frac{i_{k1}}{I_L} \longrightarrow \text{eq (2)}$

also  $d_{k2} = \frac{i_{k2}}{I_L} \longrightarrow \text{eq (3)}$

Now, if all the generators are connected to the power system simultaneously to supply the same load, by the principle of super position

$I_k = d_{k1} I_{g1} + d_{k2} I_{g2} + \dots + d_{kng} I_{ng}$

Let the individual load currents remain a constant complex ratio of the total load current  $I_L$   $\longrightarrow \text{eq (4)}$

It is assumed that  $(X/R)$  ratio for all the line elements or branches in the N/w remains the same.

The factors ' $d_{ki}$ ' will then be real & not complex. The individual generator currents may have phase angles  $\delta_1, \delta_2, \dots, \delta_{ng}$  with respect to a reference

axis. The generator currents can be expressed as:

$$i_{g1} = |i_{g1}| \cos \delta_1 + |i_{g1}| \sin \delta_1$$

(32)

$$i_{g2} = |i_{g2}| \cos \delta_2 + |i_{g2}| \sin \delta_2$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$i_{ng} = |i_{ng}| \cos \delta_{ng} + |i_{ng}| \sin \delta_{ng}$$

→ eq(5)

For simplicity to derive the formula.

Let  $n_g = 3$  in eq(4), becomes

$$I_k = d_{k1} i_{g1} + d_{k2} i_{g2} + d_{k3} i_{g3} \rightarrow \text{eq(6)}$$

$$\Rightarrow |I_k|^2 = \left( d_{k1} |i_{g1}| \cos \delta_1 + d_{k2} |i_{g2}| \cos \delta_2 + d_{k3} |i_{g3}| \cos \delta_3 \right)^2 + \left( d_{k1} |i_{g1}| \sin \delta_1 + d_{k2} |i_{g2}| \sin \delta_2 + d_{k3} |i_{g3}| \sin \delta_3 \right)^2$$

→ eq(7)

$$= d_{k1}^2 |i_{g1}|^2 \cos^2 \delta_1 + d_{k2}^2 |i_{g2}|^2 \cos^2 \delta_2 + d_{k3}^2 |i_{g3}|^2 \cos^2 \delta_3$$

$$+ 2 d_{k1} |i_{g1}| \cdot d_{k2} |i_{g2}| \cdot \cos \delta_1 \cdot \cos \delta_2 + 2 d_{k2} |i_{g2}| \cdot d_{k3} |i_{g3}| \cdot \cos \delta_2 \cdot \cos \delta_3 + 2 d_{k3} |i_{g3}| \cdot d_{k1} |i_{g1}| \cdot \cos \delta_3 \cdot \cos \delta_1 +$$

$$d_{k1}^2 |i_{g1}|^2 \sin^2 \delta_1 + d_{k2}^2 |i_{g2}|^2 \sin^2 \delta_2 + d_{k3}^2 |i_{g3}|^2 \sin^2 \delta_3 +$$

$$+ 2 d_{k1} |i_{g1}| \cdot d_{k2} |i_{g2}| \cdot \sin \delta_1 \cdot \sin \delta_2 + 2 d_{k2} |i_{g2}| \cdot d_{k3} |i_{g3}| \cdot \sin \delta_2 \cdot \sin \delta_3 + 2 d_{k3} |i_{g3}| \cdot d_{k1} |i_{g1}| \cdot \sin \delta_3 \cdot \sin \delta_1$$



$$= d_{k_1}^2 |i_{g_1}|^2 (\cos^2 \delta_1 + \sin^2 \delta_1) + d_{k_2}^2 |i_{g_2}|^2 (\cos^2 \delta_2 + \sin^2 \delta_2) + d_{k_3}^2 |i_{g_3}|^2 (\cos^2 \delta_3 + \sin^2 \delta_3) + \quad (33)$$

$$+ 2 d_{k_1} d_{k_2} |i_{g_1}| |i_{g_2}| \cos(\delta_1 - \delta_2)$$

$$+ 2 d_{k_1} d_{k_3} |i_{g_1}| |i_{g_3}| \cos(\delta_3 - \delta_1)$$

$$+ 2 d_{k_2} d_{k_3} |i_{g_2}| |i_{g_3}| \cos(\delta_2 - \delta_3)$$

$$\left. \begin{aligned} & \cos A \cos B + \sin A \sin B \\ & = \cos(A - B) \\ & \cos^2 \theta + \sin^2 \theta = 1 \end{aligned} \right\}$$

$$\Rightarrow |I_k|^2 = d_{k_1}^2 |i_{g_1}|^2 + d_{k_2}^2 |i_{g_2}|^2 + d_{k_3}^2 |i_{g_3}|^2 + 2 d_{k_1} d_{k_2} |i_{g_1}| |i_{g_2}| \cos(\delta_1 - \delta_2) + 2 d_{k_2} d_{k_3} |i_{g_2}| |i_{g_3}| \cos(\delta_2 - \delta_3) + 2 d_{k_3} d_{k_1} |i_{g_3}| |i_{g_1}| \cos(\delta_3 - \delta_1)$$

→ (8)

Eliminating currents in terms of powers supplied by the generators

$$i_{g_1} = \frac{P_1}{\sqrt{3} |V_1| \cos \phi_1}$$

$$i_{g_2} = \frac{P_2}{\sqrt{3} |V_2| \cos \phi_2}$$

$$i_{g_3} = \frac{P_3}{\sqrt{3} |V_3| \cos \phi_3}$$

where,  $P_1, P_2, P_3 =$  Active power supplied by generators 1, 2, 3 at

voltages  $|V_1|, |V_2|, |V_3|$  and power factors at the generator buses being  $\cos\phi_1, \cos\phi_2, \cos\phi_3$  respectively.

$$|I_k|^2 = \frac{d_{k1}^2 \cdot P_1^2}{(\sqrt{3} |V_1| \cos\phi_1)^2} + \frac{d_{k2}^2 \cdot P_2^2}{(\sqrt{3} |V_2| \cos\phi_2)^2} + \frac{d_{k3}^2 \cdot P_3^2}{(\sqrt{3} |V_3| \cos\phi_3)^2}$$

$$+ \frac{2d_{k1}d_{k2} \cdot P_1 P_2 \cos(\delta_1 - \delta_2)}{3 |V_1| |V_2| \cos\phi_1 \cdot \cos\phi_2} + \frac{2d_{k2}d_{k3} \cdot P_2 P_3 \cos(\delta_2 - \delta_3)}{3 |V_2| |V_3| \cos\phi_2 \cos\phi_3}$$

$$+ \frac{2d_{k3}d_{k1} \cdot P_3 P_1 \cos(\delta_3 - \delta_1)}{3 |V_3| |V_1| \cos\phi_3 \cos\phi_1} \longrightarrow (8)$$

The power losses in the network comprising of  $nb - nb_0$  elements or branches  $P_{Loss}$  is given by

$$P_L = \sum_{k=1}^{nb} 3 |i_k|^2 R_k \longrightarrow (9)$$

where,  $R_k =$  resistance of the element  $k$ ,

sub eq (8) in eq (9) we get.

$\Rightarrow P_{Loss} =$

i.e  $|I_k|^2$  value in power loss,  $P_L$ .

$P_{Loss} =$



$$\begin{aligned}
 P_L = & \frac{P_1^2 \sum_{k=1}^{ng} d_{k1}^2 R_k}{|V_1|^2 \cos^2 \phi_1} + \frac{P_2^2 \sum_{k=1}^{ng} d_{k2}^2 R_k}{|V_2|^2 \cos^2 \phi_2} + \frac{P_3^2 \sum_{k=1}^{ng} d_{k3}^2 R_k}{|V_3|^2 (\cos \phi_3)^2} + \\
 & + \frac{2 P_1 P_2 \sum_{k=1}^{ng} d_{k1} d_{k2} R_k \cos(\delta_1 - \delta_2)}{|V_1| |V_2| \cos \phi_1 \cdot \cos \phi_2} + \\
 & + \frac{2 P_2 P_3 \sum_{k=1}^{ng} d_{k2} d_{k3} R_k \cos(\delta_2 - \delta_3)}{|V_2| |V_3| \cos \phi_2 \cos \phi_3} \\
 & + \frac{2 P_3 P_1 \sum_{k=1}^{ng} d_{k3} d_{k1} R_k \cos(\delta_3 - \delta_1)}{|V_3| |V_1| \cos \phi_3 \cdot \cos \phi_1} \longrightarrow \text{eq (10)}
 \end{aligned}$$

Let  $B_{11} = \frac{1}{|V_1|^2 (\cos \phi_1)^2} \sum d_{k1}^2 R_k \longrightarrow (11)$

$B_{22} = \frac{1}{|V_2|^2 (\cos \phi_2)^2} \sum d_{k2}^2 R_k \longrightarrow \text{eq (12)}$

$B_{33} = \frac{1}{|V_3|^2 (\cos \phi_3)^2} \sum d_{k3}^2 R_k \longrightarrow \text{eq (13)}$

$B_{12} = \frac{\cos(\delta_1 - \delta_2) \sum_{k=1}^{ng} d_{k1} d_{k2} R_k}{|V_1| |V_2| (\cos \phi_1) (\cos \phi_2)} \longrightarrow \text{eq (14)}$

$B_{23} = \frac{\cos(\delta_2 - \delta_3) \sum_{k=1}^{ng} d_{k2} d_{k3} R_k}{|V_2| |V_3| (\cos \phi_2) (\cos \phi_3)} \longrightarrow \text{eq (15)}$

$$B_{31} = \frac{\cos(\delta_3 - \delta_1) \sum_{k=1}^{n_g} d_{k3} d_{k1} R_k}{|V_1| |V_3| (\cos \phi_3) (\cos \phi_1)} \rightarrow \text{eq (16)}$$

$$\Rightarrow P_{\text{Loss}} = P_1^2 B_{11} + P_2^2 B_{22} + P_3^2 B_{33} + 2 P_1 P_2 B_{12} + 2 P_2 P_3 B_{23} + 2 P_3 P_1 B_{31} + \dots \rightarrow \text{eq (17)}$$

$$\Rightarrow P_{\text{Loss}} = \sum_{m=1}^3 \sum_{n=1}^3 P_m B_{mn} P_n \rightarrow \text{eq (18)}$$

In general the formula for  $B_{mn}$  coefficient can be expressed as

$$B_{mn} = \frac{\cos(\delta_m - \delta_n)}{|V_m| |V_n| (\cos \phi_m) (\cos \phi_n)} \cdot \sum_k d_{km} d_{kn} R_k \rightarrow \text{eq (19)}$$

eq (18) is called General transmission loss formula

and in the matrix form it can be represented

as.

$$P_{\text{Loss}} = \begin{bmatrix} P_1 & P_2 & \dots & P_n \end{bmatrix}_{1 \times n} \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \vdots & \vdots & \dots & \vdots \\ B_{n1} & B_{n2} & \dots & B_{nn} \end{bmatrix}_{n \times n} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix}_{n \times 1} \rightarrow \text{eq (20)}$$



## Problems :-

UNIT - T

(1)

1. The fuel input per hour of plants 1 & 2 are given as

$$F_1 = 0.2P_1^2 + 40P_1 + 120 \text{ Rs/hr}$$

$$F_2 = 0.25P_2^2 + 30P_2 + 150 \text{ Rs/hr.}$$

Determine the economic operating schedule and the corresponding cost of generation if the maximum & minimum loading on each unit is 100 MW & 25 MW, the demand is 180 MW, & transmission losses neglected. If the load is equally shared by both the units, determine the saving obtained by loading the units as per equal incremental production cost.

Sol: - The incremental production cost of both the units

are

$$\frac{dF_1}{dP_1} = 0.4P_1 + 40 \text{ Rs/MWhr}$$

$$\& \frac{dF_2}{dP_2} = 0.5P_2 + 30 \text{ Rs/MWhr.}$$

Now for economic operation of the units

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$\therefore 0.4P_1 + 40 = 0.5P_2 + 30 \rightarrow \text{eq (1)}$$

$$\& P_1 + P_2 = 180 \rightarrow \text{eq (2)}$$

solution of these equations,

$$P_1 = 88.89 \text{ MW}$$

$$P_2 = 91.11 \text{ MW}$$

Now the cost of generation =  $F_1 + F_2$

(2)

$$F_1 = 0.2P_1^2 + 40P_1 + 120 = 0.2(88.89)^2 + 40(88.89) + 120 \\ = 5255.88 \text{ Rs/hx}$$

$$F_2 = 0.25P_2^2 + 30P_2 + 150 = 0.25(91.11)^2 + 30(91.11) + 150 \\ = 4958.55 \text{ Rs/hx}$$

$$\text{total cost} = 5255.88 + 4958.55 = 10214.43 \text{ Rs/hx}$$

b) If the load on each unit is 90MW, the cost of generation will be

$$F_1 = 5340 \text{ Rs/hx}$$

$$F_2 = 4895 \text{ Rs/hx}$$

$$\therefore \text{total cost} = 10215 \text{ Rs/hx}$$

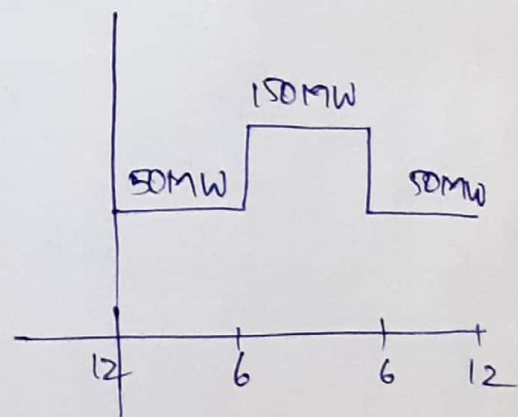
$$\text{Saving will be} = 10215 - 10214.43 = 0.57 \text{ Rs/hx}$$

2) Assume that the fuel input in BTU/hx for units 1 and 2 are given by

$$C_1 = (8P_{G1} + 0.024P_{G1}^2 + 80) \times 10^6$$

$$C_2 = (6P_{G2} + 0.04P_{G2}^2 + 120) \times 10^6$$

The maximum and minimum loads on the units are 100MW & 10MW respectively.



Determine the minimum cost of generation when the following load shown in fig is supplied. The cost of fuel is 2 Rs/million BTU.

Sol:  $\frac{dC_1}{dP_{G1}} = 0.048P_{G1} + 8$



$$C_2 = (6 P_{G2} + 0.04 P_{G2}^2 + 120) \text{ MBTU/hr} \quad (3)$$

$$\frac{dC_2}{dP_{G2}} = 0.08 P_{G2} + 6$$

a) when load is 50 MW, for economic loading the conditions are  $\frac{dC_1}{dP_{G1}} = \frac{dC_2}{dP_{G2}} = \lambda$ .

$$\Rightarrow P_{G1} + P_{G2} = 50 \longrightarrow (1)$$

$$\therefore 0.048 P_{G1} + 8 = 0.08 P_{G2} + 6$$

$$0.048 P_{G1} - 0.08 P_{G2} + 2 = 0 \longrightarrow \text{eq(2)}$$

solving eq (1) & (2) i.e.  $P_{G1} = 50 - P_{G2}$

$$\Rightarrow 0.048(50 - P_{G2}) - 0.08 P_{G2} + 2 = 0$$

$$2.4 - 0.048 P_{G2} - 0.08 P_{G2} + 2 = 0$$

$$4.4 - 0.128 P_{G2} + 2 = 0$$

$$6.4 = 0.128 P_{G2}$$

$$\Rightarrow P_{G2} = \frac{6.4}{0.128} = 50 \text{ MW}$$

$$\Rightarrow P_{G1} = 50 - P_{G2} = 0 \text{ MW}$$

$$\Rightarrow C_1 = (8(15.625) + 0.024(15.625)^2 + 80) \text{ million BTU/hr}$$

$$C_1 = 210.868 \text{ M BTU/hr}$$

$$C_2 = 373.5 \text{ M BTU/hr}$$

b) when the load is 150 MW

$$0.048 P_{G1} - 0.08 P_{G2} + 2 = 0 \longrightarrow \text{eq(2)}$$

$$P_{G1} + P_{G2} = 150$$

(4)

$$\Rightarrow \begin{cases} P_{G1} = 78.126 \text{ MW} \\ P_{G2} = 71.874 \text{ MW} \end{cases}$$

$$\Rightarrow \begin{cases} C_1 = 851.496 \text{ MBTU/hr} \\ C_2 = 757.87 \text{ MBTU/hr} \end{cases}$$

therefore;  $\text{Cost} = \text{Rs} (210.868 + 378.5 + 851.496 + 757.87) \times 12 \times 2$

$$\therefore \text{Cost} = \text{Rs} \cdot 52649.61$$

3) The fuel input per hour of plant-1 & 2 are given as  $C_1 = 0.2 P_{G1}^2 + 40 P_{G1} + 120 \text{ Rs/hr}$

$$C_2 = 0.25 P_{G2}^2 + 30 P_{G2} + 150 \text{ Rs/hr}$$

Determine the economic operating schedule and the corresponding cost of generation, if the maximum & minimum loading on each unit is 100 MW & 25 MW, the demand is 180 MW and transmission losses are neglected. If the load is equally shared by both the units, determine the savings obtained by loading the units as per equal incremental production cost.

Sol:- a)  $P_{G1} + P_{G2} = 180 \Rightarrow P_{G1} = 88.89 \text{ MW}, P_{G2} = 91.11 \text{ MW}$

$$C_1 = 5255.88 \text{ Rs/hr}, C_2 = 4958.55 \text{ Rs/hr}$$

$$\text{Total cost}_1 = \text{Rs} \cdot 10214.43/\text{hr}$$

b)  $P_{G1} + P_{G2} = 90 \text{ MW}$

$$\Rightarrow C_1 = \text{Rs} \cdot 5340/\text{hr}$$

$$C_2 = \text{Rs} \cdot 4875/\text{hr}$$

$$\text{Total cost}_2 = 10215 \text{ Rs/hr}$$

$$\begin{aligned} \text{Total saving} &\geq \\ \text{Total cost}_2 - \text{Total cost}_1 &= 10215 - 10214.43 \\ &= \text{Rs} \cdot 0.57/\text{hr} \end{aligned}$$



5) The cost curves of two plants are

$$C_1 = 0.05 P_{G1}^2 + 20 P_{G1} + 150 \text{ Rs/h}$$

$$C_2 = (0.05 P_{G2}^2) + 15 P_{G2} + 180 \text{ Rs/h}$$

The loss coefficient for the above system is given

as  $B_{11} = 0.0015 / \text{MW}$  ;  $B_{12} = -0.0004 / \text{MW}$ . Determine

the economical generation scheduling corresponding to  $\lambda = 25 \text{ Rs/MWh}$  and the corresponding system load that can be met with. If the <sup>total</sup> load connected to the system is 120 MW. taking 4% change in the value of  $\lambda$ , what should be the value of ' $\lambda$ ' in the next iteration

Sol: - Give that the cost curves of two plants are

$$C_1 = 0.05 P_{G1}^2 + 20 P_{G1} + 150 \text{ Rs/h}$$

The incremental costs are

$$\frac{dC_1}{dP_{G1}} = 0.1 P_{G1} + 20 \text{ Rs/MWh}$$

$$\frac{dC_2}{dP_{G2}} = 0.1 P_{G2} + 15 \text{ Rs/MWh}$$

Transmission loss,  $P_L = \sum_{p=1}^n \sum_{q=1}^n P_{Gp} B_{pq} P_{Gq}$

For two plants,  $n=2$  & we have

$$P_L = \sum_{m=1}^2 \sum_{n=1}^2 P_{Gm} B_{mn} P_{Gn}$$

$$= P_{G1} B_{11} P_{G1} + P_{G1} B_{12} P_{G2} + P_{G2} B_{21} P_{G1} + P_{G2} B_{22} P_{G2}$$

$$= 0.0015 P_{G1}^2 + 2(-0.0004) P_{G1} P_{G2} + 0.0032 P_{G2}^2$$

$$P_L = 0.0015 P_{G1}^2 - 0.0008 P_{G1} P_{G2} + 0.0032 P_{G2}^2$$

→ eq (1)

the ITL of plant-1 is

(6)

$$\begin{aligned} (ITL)_1 &= \frac{\partial P_L}{\partial P_{G1}} = 2(0.0015)P_{G1} - 0.0008P_{G2} \\ &= 0.003P_{G1} - 0.0008P_{G2} \longrightarrow (2) \end{aligned}$$

the ITL of plant-2 is

$$(ITL)_2 = \frac{\partial P_L}{\partial P_{G2}} = -0.0008P_{G1} + 0.0064P_{G2} \longrightarrow \text{eq(3)}$$

the penalty factor of plant-1 is

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}} = \frac{1}{1 - (0.003P_{G1} - 0.0008P_{G2})} \longrightarrow \text{eq(4)}$$

and the penalty factor of plant-2 is

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G2}}} = \frac{1}{1 - (-0.0008P_{G1} + 0.0064P_{G2})} \longrightarrow \text{eq(5)}$$

the condition for optimum operation is

$$\frac{\partial C_1}{\partial P_{G1}} \cdot L_1 = \frac{\partial C_2}{\partial P_{G2}} \cdot L_2 = \lambda$$

$$\frac{\partial C_1}{\partial P_{G1}} \cdot L_1 = \lambda$$

$$\Rightarrow (0.01P_{G1} + 20) \frac{1}{1 - (0.003P_{G1} - 0.0008P_{G2})} = 30$$

$$0.01P_{G1} + 20 = 30[1 - (0.003P_{G1} - 0.0008P_{G2})]$$

$$(08) \quad 1.69P_{G1} - 0.024P_{G2} = 10 \longrightarrow \text{eq(6)}$$

$$\text{and } \frac{\partial C_2}{\partial P_{G2}} \cdot L_2 = \lambda \longrightarrow \text{eq(7)}$$



$$(0.1 P_{G2} + 15) \frac{1}{1 - (-0.0008 P_{G1} + 0.0064 P_{G2})} = 30$$

$$(0.1 P_{G2} + 15) = 30 [1 - (-0.0008 P_{G1} + 0.0064 P_{G2})]$$

(os)  $0.024 P_{G1} - 0.292 P_{G2} = -15 \rightarrow \text{eq(8)}$

solving eq(6) & eq(8) we get

$$1.09 P_{G1} - 0.024 P_{G2} = 10 \rightarrow \text{eq(6)} \times 0.024$$

$$0.024 P_{G1} - 0.292 P_{G2} = -15 \rightarrow \text{eq(8)} \times 1.09$$

---


$$\begin{array}{r} 0.02616 P_{G1} - 0.000576 P_{G2} = 0.24 \\ 0.02616 P_{G1} - 0.31828 P_{G2} = -16.36 \\ \hline \end{array}$$

$$0.3177 P_{G2} = 16.6 \Rightarrow \boxed{P_{G2} = 52.25 \text{ MW}}$$

Substituting  $P_{G2}$  in eq(6) we get,  $1.09 P_{G1} - 0.024(52.25) = 10$

$$\therefore \boxed{P_{G1} = 10.325 \text{ MW}}$$

Transmission loss,  $P_L = 0.0015 (10.325)^2 - 0.0008 (10.325)(52.25) + 0.0032 (52.25)^2$

$$\therefore \boxed{P_L = 8.465 \text{ MW}}$$

Corresponding system load,  $P_D = P_{G1} + P_{G2} - P_L$

$$\Rightarrow P_D = 10.325 + 52.25 - 8.465 = 54.11 \text{ MW}$$

For 4% change in value of  $\lambda$ ,  $\Delta \lambda = 4\%$  of  $30 = 1.2 \text{ Rs/MWh}$

New load connected to system,  $P_D = 120 \text{ MW}$

(8)

$$\therefore \text{change in load, } \Delta P_D = 120 - 54.11 = 65.89 \text{ MW}$$

Hence, change in load,  $\Delta P_D > 0$ ; hence, to get an optimum dispatch decrease  $\lambda$  by  $\Delta \lambda$ .

$$\begin{aligned} \text{New value of } \lambda = \lambda' &= \lambda - \Delta \lambda = 30 - 1.2 \\ &= 28.8 \text{ Rs/MWh} // \end{aligned}$$



HYDRO THERMAL SCHEDULINGIntroduction :-

No state or country is endowed with plenty of water sources or abundant coal & Nuclear fuel. For minimum environmental pollution, thermal generation should be minimum.

Hence a mix of hydro and thermal power generation is necessary. The states that have a large hydro-potential can supply excess hydro-power during periods of high water run-off to other states.

The states which have a low hydro-potential & large coal reserves, can use the small hydro-power for meeting peak load requirements. This makes the thermal stations to operate at high load factors and to have reduced installed capacity with the result economy.

In states, which have adequate hydro as well as thermal-power generation capacities, power co-ordination to obtain a most economical operating state is essential.

OPTIMAL SCHEDULING OF HYDRO THERMAL SYSTEM :-HYDRO ELECTRIC POWER PLANT MODELS :-

Hydro electric power plants harness water power for generation of electric energy. The potential energy of water is converted to kinetic energy. When water drops through a height the energy is able to rotate turbines which are coupled to alternators. These plants have the advantage of very low operating cost. Moreover they can be started and loaded

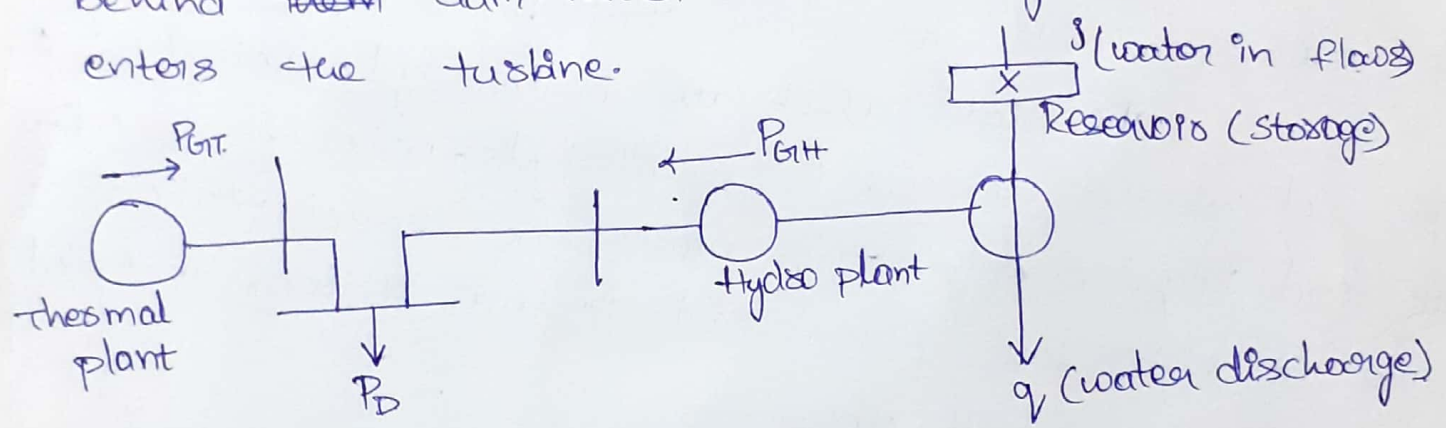


quickly. However they take long time for installation and entail huge investment. Moreover, their first capacity is low and need to be backed by steam plants

1) classification :-

a) Run off river plant :- they use water as it comes. No storage is required. Evidently there is no control on flow of water.

b) Reservoir plants :- water is stored in a big reservoir behind ~~them~~ dam. water flows through penstock and enters the turbine.



Fig(1) :- Fundamental hydrothermal system.

operation of a system having both hydro and thermal plants is however far more complex. As hydro plants have negligible operating cost but are required to operate under constraints of water available for hydro generation in a given period of time.

The whole or a part of the base load can be ~~conserved~~ supplied by the run-off river hydro plants, and the peak or the remaining load is then met by a proper mix of reservoir-type hydro plants and thermal plants. Determination of this by a proper



mix is the determination of the most economical (3) operating state of a hydro-thermal system. The hydro-thermal co-ordination is classified into long-term-co-ordination & short-term co-ordination.

2) Types of turbines :-

- a) Pelton turbine :- It is characterised by high head and low quantity of water.
- b) Francis turbine :- It is a reaction turbine suitable for medium heads & medium water flow.
- c) Kaplan turbine :- It is also a reaction turbine suitable for low head & large quantity of water.

3) Pumped storage plants :-

It is a special type of plant suitable for supplying peak load. During peak load, it generates electrical energy. During off-peak period the same water is pumped back from tail water pond to the head water pond so that the same water is used again to generate electrical energy.

SCHEDULING PROBLEMS :-

The whole or a part of the base load can be supplied by the run-off river hydro-plants, and the peak or the remaining load is then met by a proper mix of reservoir-type hydro-plants and thermal plants.

Determination of this by a proper mix is the determination of the most economical operating state of a hydro-thermal system. The hydro-thermal co-ordination is classified into

- 1) Long-term co-ordination
- 2) Short-term co-ordination

1) Long-term co-ordination :-

typical long-term co-ordination may be extended from one week to one year or several years. The co-ordination of the operation of reservoir hydro-power plants and steam plants involves the best utilization of available water in terms of the scheduling of water released.

In other words, since the operating costs of hydro-plants are very low, hydro-power can be generated at very little incremental cost.

In a combined operational system, the generation of thermal power should be displaced by available hydro-power so that maximum decrement production costs will be realised at the steam plant.

The long-term scheduling problem involves the long-term forecasting of water availability and the scheduling of reservoir water releases for an interval of time that depends on the reservoir capacities and the chronological load curve of the system. Based on these factors during different times of the year, the hydro and steam plants can be operated as base load plants and peak load plants and vice versa.

Long term scheduling is made based on an optimising policy in view of statistically treated unknown



such as load, hydraulic inflows, and unit availability (5)  
(i.e. steam and hydro plants).

The useful techniques employed for this type of scheduling problems include.

- i) The simulation of an entire long-term operational time period for a given set of operating conditions by using the dynamic programming method.
- ii) Composite hydraulic simulation models, and
- iii) Statistical production cost models.

For the long-term scheduling of a hydro-thermal system, there should be required generation to meet the requirements of load demand and both hydro and thermal generations should be so scheduled so as to maintain the minimum fuel costs. This requires that the available water should be put to an optimum use.

### Mathematical Formulation of Long-term Hydro-thermal scheduling :-

In order to have the mathematical formulation of optimal scheduling problem in a hydro-thermal system, the following assumptions are to be made for a certain period of operation  $T$  (a day, a week, or a year)

- i) The storage of a hydro-reservoir at the beginning and at the end of period of operation ' $T$ ' are specified.
- ii) After accounting for the irrigation purpose, water inflow to the reservoir and load demand on the system

are known deterministically as functions of time (6) with certainties.

The optimization problem here is to determine the water discharge rate ' $q(t)$ ' so as to minimize the cost of thermal generation.

Objective function is

$$\min C_T = \int_0^T c' [P_{GT}(t)] dt \longrightarrow \text{eq(1)}$$

Subject to the following constraints:

i) the real power balance equation

$$P_{GT}(t) + P_{GH}(t) = P_L(t) + P_D(t)$$

i.e. 
$$P_{GT}(t) + P_{GH}(t) - P_L(t) - P_D(t) = 0 \quad \text{for } t \in (0, T)$$
  $\longrightarrow \text{eq(2)}$

where

$P_{GT}(t)$  = Real power thermal generation at time ' $t$ '

$P_{GH}(t)$  = Real power hydro generation at time ' $t$ '

$P_L(t)$  = real power loss at time ' $t$ ', and

$P_D(t)$  = real power demand at time ' $t$ '.

ii) water availability equation:

$$x'(t) - x'(0) - \int_0^T J(t) dt + \int_0^T q(t) dt = 0 \longrightarrow \text{eq(3)}$$

where,  $x'(t)$  = the water storage at time ' $t$ '

$x'(0)$  = the water storage at the beginning of operation time,  $T$ .



$J(t)$  = the water inflow rate and  
 $q(t)$  = the water discharge rate.

(7)

### ii) Real power hydro generation :-

The real power hydro-generation  $P_{GH}(t)$  is a function of water storage  $x'(t)$  and water discharge rate  $q(t)$ .

i.e. 
$$P_{GH}(t) = f[x'(t), q(t)]$$

### 2) SHORT-TERM HYDROTHERMAL COORDINATION :-

The economic operation of an all steam system depends only on the conditions that exist from instant to instant. However the economic operation of a combined hydrothermal system depends on the conditions existing over the whole of the operating period.

The problem is how to supply load, as per load cycle during the operating period (say one day). The factors on which the economic operation of a combined hydro-thermal system depends are.

- 1) Load cycle
- 2) Incremental fuel costs of thermal power stations.
- 3) Expected water inflow in hydro power stations.
- 4) Water head which is a function of water storage in hydro power stations.
- 5) Hydro power generation
- 6) Incremental transmission loss.

The objective of hydro-thermal co-ordination in a combined system is to supply power to the load as per

the load cycle during the operating period (for eg: a 24 hrs) (8)  
such that the cost of thermal power is minimum and the water used for hydro-power generation is a certain desired quantity at a constant head over this period. A few important methods of hydro-thermal co-ordination are described below.

1) constant hydro generation method :- In this, the hydro power generation is kept constant throughout the operating period at such a value as to use a scheduled amount of water at a constant head. The remaining load is met by the thermal plants.

2) constant hydro thermal generation method :-  
Thermal generation is kept constant, ~~see~~ throughout the operating period in such a way that the hydro power plants use the specified or scheduled amount of water and operate on varying power generation schedule during the operating period.

3) Maximum hydro efficiency method :-  
In this, hydro power plants are operated at maximum efficiency during peak load periods and as near the maximum efficiency as possible during the off-peak load periods. During off-peak load periods, specified amount of water is used for hydro power generation.

4) KIRCHMAYER'S METHOD :- (8) mathematical formulation of short-term co-ordination :-



## KIRCHMAYER'S METHOD (short-term hydro-thermal coordination) 9

In this method, the co-ordination equations are derived in terms of penalty factors of both plants for obtaining the optimum scheduling of a hydro-thermal system and hence it is also known as the "Penalty factor method" of solution of short-term hydro-thermal scheduling problems.

Let  $P_{GT_i}$  = the power generation of  $i$ th thermal plant in MW,

$P_{GH_j}$  = the power generation of  $j$ th hydro plant in MW,

$\frac{dc_i}{dP_{GT_i}}$  = the incremental fuel cost of  $i$ th thermal plant in Rs/MWh.

$W_j$  = the quantity of water used for power generation at  $j$ th hydro-plant in  $m^3/s$

$\frac{dW_j}{dP_{GH_j}}$  = the incremental water rate of  $j$ th hydro-plant in  $m^3/s/MW$ ,

$\frac{\partial PL}{\partial P_{GT_i}}$  = the incremental transmission loss of  $i$ th thermal plant,

$\frac{\partial PL}{\partial P_{GH_j}}$  = the incremental transmission loss of  $j$ th hydro plant.

$\lambda$  = Lagrangian multiplier,

$\delta_j$  = the constant which converts the incremental water rate of hydro plant ' $j$ ' into an incremental cost,

$n$  = the total number of plants

$\alpha$  = the number of thermal plants

$(n - \alpha)$  = the number of hydro-plants, &

$\tau$  = time interval during which the plant operation is considered.

Here, the objective is to find the generation of individual plants, both thermal as well as hydro that the generation cost (cost of fuel in thermal) is optimum and at the same time total demand ( $P_D$ ) and losses ( $P_L$ ) are continuously met.

As it is short-range problem, there will not be any appreciable change in the level of water in the reservoirs during the interval (i.e., the effects of rainfall and evaporation are neglected) and hence the head of water in the reservoir will be assumed to be constant.

Let  $k_j$  = specified quantity of water, which must be utilized within the interval  $\tau$  at each hydro-station  $j$ .

Problem formulation:-

The objective function is to minimize the cost of generation:

$$\text{i.e. } \min \sum_{i=1}^{\alpha} \int_0^{\tau} C_i dt \quad \longrightarrow \text{eq(1)}$$

Subject to the equality constraints

$$\sum_{i=1}^{\alpha} P_{GHi} + \sum_{j=\alpha+1}^n P_{GHyj} = P_D + P_L \quad \longrightarrow \text{eq(2)}$$

$$\text{and } \int_0^{\tau} w_j dt = k_j \quad \text{for } j = \alpha+1, \alpha+2, \dots, n$$

$\longrightarrow \text{eq(3)}$

where,  $w_j$  = the turbine discharge in the  $j$ th plant in  $m^3/s$   
 $k_j$  = the amount of water in  $m^3$  utilized during the time period  $\tau$  in the  $j$ th hydro-plant.

The coefficient 'g' must be selected so as to use the



specified amount of water during the operating period. Now, the objective function becomes

$$\min C = \sum_{i=1}^{\alpha} \int_0^T c_i dt + \sum_{j=\alpha+1}^n \delta_j k_j$$

substituting  $k_j$  from eq(3) in the above eqn, we get

$$\min C = \sum_{i=1}^{\alpha} \int_0^T c_i dt + \sum_{j=\alpha+1}^n \delta_j \int_0^T w_j dt \quad \rightarrow \text{eq(4)}$$

For a particular load demand  $P_D = \text{const}$ , eq(2) results as

$$\sum_{i=1}^{\alpha} \Delta P_{GTi} + \sum_{j=\alpha+1}^n \Delta P_{G Hj} - \sum_{i=1}^{\alpha} \frac{\partial P_L}{\partial P_{GTi}} \Delta P_{GTi} - \sum_{j=\alpha+1}^n \frac{\partial P_L}{\partial P_{G Hj}} \Delta P_{G Hj} = 0$$

For a particular hydro-plant  $x$ , eq(5) can be rewritten as

$$\Delta P_{G Hx} - \frac{\partial P_L}{\partial P_{G Hx}} \Delta P_{G Hx} - \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \Delta P_{G Hj} + \sum_{i=1}^{\alpha} \frac{\partial P_L}{\partial P_{GTi}} \Delta P_{GTi} + \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \frac{\partial P_L}{\partial P_{G Hj}} \Delta P_{G Hj} = 0$$

By rearranging the above equation, we get

$$\left[ 1 - \frac{\partial P_L}{\partial P_{G Hx}} \right] \Delta P_{G Hx} = - \sum_{i=1}^{\alpha} \left[ 1 - \frac{\partial P_L}{\partial P_{GTi}} \right] \Delta P_{GTi} - \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \left[ 1 - \frac{\partial P_L}{\partial P_{G Hj}} \right] \Delta P_{G Hj}$$

From eq(4), the condition for minimization is  $\rightarrow \text{eq(6)}$

$$\Delta \left[ \sum_{i=1}^{\alpha} \int_0^T c_i dt + \sum_{j=\alpha+1}^n \delta_j \int_0^T w_j dt \right] = 0 \quad \rightarrow \text{eq(7)}$$

The above equation can be written as  $\sum_{i=1}^{\alpha} c_i + \sum_{j=\alpha+1}^n \delta_j w_j = 0 \Rightarrow \frac{d}{dt} \left[ \sum_{i=1}^{\alpha} c_i + \sum_{j=\alpha+1}^n \delta_j w_j \right] = 0$

$$\sum_{i=1}^{\alpha} \frac{dc_i}{dP_{GTi}} \Delta P_{GTi} + \sum_{j=\alpha+1}^n \delta_j \frac{dw_j}{dP_{GHj}} \Delta P_{GHj} = 0 \quad \longrightarrow \text{eq(8)}$$

For hydro-plant x,

$$\delta_x \frac{dw_x}{dP_{GHx}} \Delta P_{GHx} = - \sum_{i=1}^{\alpha} \frac{dc_i}{dP_{GTi}} \Delta P_{GTi} - \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \delta_j \frac{dw_j}{dP_{GHj}} \Delta P_{GHj}$$

Multiplying the above equation by  $\left[1 - \frac{\partial P_L}{\partial P_{GHx}}\right]$ ,

$$\left[1 - \frac{\partial P_L}{\partial P_{GHx}}\right] \delta_x \frac{dw_x}{dP_{GHx}} \Delta P_{GHx} = \left[1 - \frac{\partial P_L}{\partial P_{GHx}}\right] \left[ - \sum_{i=1}^{\alpha} \frac{dc_i}{dP_{GTi}} \Delta P_{GTi} - \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \delta_j \frac{dw_j}{dP_{GHj}} \Delta P_{GHj} \right] \quad \longrightarrow \text{eq(9)}$$

Substituting the values of  $\left[1 - \frac{\partial P_L}{\partial P_{GHx}}\right] \Delta P_{GHx}$  from eq(6) in eq(9) we get

$$\delta_x \frac{dw_x}{dP_{GHx}} \left[ - \sum_{i=1}^{\alpha} \left(1 - \frac{\partial P_L}{\partial P_{GTi}}\right) \Delta P_{GTi} - \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \left(1 - \frac{\partial P_L}{\partial P_{GHj}}\right) \Delta P_{GHj} \right] = \left[1 - \frac{\partial P_L}{\partial P_{GHx}}\right] \left[ - \sum_{i=1}^{\alpha} \frac{dc_i}{dP_{GTi}} \Delta P_{GTi} - \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \delta_j \frac{dw_j}{dP_{GHj}} \Delta P_{GHj} \right]$$

Resolving the above equation as

$$\begin{aligned} \delta_x \frac{dw_x}{dP_{GHx}} \left[ \sum_{i=1}^{\alpha} \left(1 - \frac{\partial P_L}{\partial P_{GTi}}\right) \Delta P_{GTi} - \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \left(1 - \frac{\partial P_L}{\partial P_{GHj}}\right) \Delta P_{GHj} \right] \\ + \sum_{i=1}^{\alpha} \frac{dc_i}{dP_{GTi}} \left[1 - \frac{\partial P_L}{\partial P_{GHx}}\right] \Delta P_{GTi} + \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \delta_j \frac{dw_j}{dP_{GHj}} \left[1 - \frac{\partial P_L}{\partial P_{GHx}}\right] \Delta P_{GHj} = 0 \end{aligned}$$



$$\Rightarrow \sum_{i=1}^{\alpha} \left[ \frac{dc_i}{dP_{GT_i}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_i}} \right] - \delta_x \frac{dw_x}{dP_{GT_x}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_i}} \right] \right] \Delta P_{GT_i} +$$

$$+ \left[ \sum_{\substack{j=\alpha+1 \\ j \neq x}}^n \delta_j \frac{dw_j}{dP_{GT_j}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_j}} \right] - \delta_x \frac{dw_x}{dP_{GT_x}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_j}} \right] \right] \Delta P_{GT_j} = 0$$

→ eq (10)

∴  $\Delta P_{GT_i} \neq 0$  &  $\Delta P_{GT_j} \neq 0$ , eq (10) becomes

$$\Rightarrow \frac{dc_i}{dP_{GT_i}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_x}} \right] - \delta_x \frac{dw_x}{dP_{GT_x}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_i}} \right] = 0 \quad \text{for } i=1, 2, \dots, \alpha$$

→ eq (11)

and

$$\delta_j \frac{dw_j}{dP_{GT_j}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_x}} \right] - \delta_x \frac{dw_x}{dP_{GT_x}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_j}} \right] = 0 \quad \text{for}$$

$j = \alpha+1, \alpha+2, \dots, n$

→ eq (12)

eq (11) & eq (12) can be re-written as

$$\frac{dc_i}{dP_{GT_i}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_x}} \right] = \delta_x \frac{dw_x}{dP_{GT_x}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_i}} \right]$$

$$\Rightarrow \boxed{\frac{dc_i}{dP_{GT_i}} \left[ \frac{1}{1 - \frac{\partial P_L}{\partial P_{GT_i}}} \right] = \delta_x \frac{dw_x}{dP_{GT_x}} \left[ \frac{1}{1 - \frac{\partial P_L}{\partial P_{GT_x}}} \right]}$$

→ eq (13)

$$\text{eq (12)} \Rightarrow \delta_j \frac{dw_j}{dP_{GT_j}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_x}} \right] = \delta_x \frac{dw_x}{dP_{GT_x}} \left[ 1 - \frac{\partial P_L}{\partial P_{GT_j}} \right]$$

$$\delta_j \frac{dw_j}{dP_{GHj}} \frac{1}{\left[1 - \frac{\partial PL}{\partial P_{GHj}}\right]} = \delta_x \frac{dw_x}{dP_{GHx}} \frac{1}{\left[1 - \frac{\partial PL}{\partial P_{GHx}}\right]}$$

from eq (13) & eq (14) we have → eq (14)

$$\frac{dc_i}{dP_{GTi}} \frac{1}{\left[1 - \frac{\partial PL}{\partial P_{GTi}}\right]} = \delta_j \frac{dw_j}{dP_{GHj}} \frac{1}{\left[1 - \frac{\partial PL}{\partial P_{GHj}}\right]} = \lambda$$

$$\frac{dc_i}{dP_{GTi}} \frac{1}{\left[1 - \frac{\partial PL}{\partial P_{GTi}}\right]} = (I_c)_i L_i = \lambda \quad \text{for } i=1, 2, \dots, \alpha$$
→ eq (15)

$$\delta_j \frac{dw_j}{dP_{GHj}} \frac{1}{\left[1 - \frac{\partial PL}{\partial P_{GHj}}\right]} = \delta_j (I_w)_j L_j = \lambda \quad \text{for } j=\alpha+1, \alpha+2, \dots, N$$
→ eq (16)

where,  $(I_c)_i$  = the incremental fuel cost of the  $i$ th thermal plant  
 $(I_w)_j$  = the incremental water rate of the  $j$ th hydro-plant.

eq (15) & eq (16) may be expressed as approximately as

$$\therefore \frac{dc_i}{dP_{GTi}} \left[1 + \frac{\partial PL}{\partial P_{GTi}}\right] = \lambda \quad \text{for } i=1, 2, \dots, \alpha$$
→ eq (17)

$$\delta_j \frac{dw_j}{dP_{GHj}} \left[1 + \frac{\partial PL}{\partial P_{GHj}}\right] = \lambda \quad \text{for } j=\alpha+1, \alpha+2, \dots, N$$
→ eq (18)

where,  $\left[1 + \frac{\partial PL}{\partial P_{GTi}}\right]$  and  $\left[1 + \frac{\partial PL}{\partial P_{GHj}}\right]$  are the approximate penalty



factors of the  $i$ th thermal plant and the  $j$ th hydro-plant respectively.

eq (17) & eq (18) are the co-ordinate equations, which are used to obtain the optimal scheduling of the hydro-thermal system when considering the transmission losses.

In the above equations, the transmission loss  $P_L$  is

expressed as

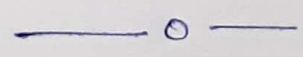
$$P_L = \sum_{i=1}^{\alpha} \sum_{k=1}^{\alpha} P_{GTi} B_{ik} P_{GTk} + \sum_{g=\alpha+1}^n \sum_{j=\alpha+1}^n P_{GHg} B_{gj} P_{GHj} + 2 \sum_{i=1}^{\alpha} \sum_{j=\alpha+1}^n P_{GTi} B_{ij} P_{GHj}$$

→ eq (19)

The power generation of a hydro plant  $P_{GHj}$  is directly proportional to its head discharge rate  $w_j$

when neglecting the transmission losses, the coordination equations become

$$\boxed{\frac{\partial C_i}{\partial P_{GTi}} = \lambda} \quad ; \quad \boxed{\lambda_j \frac{\partial w_j}{\partial P_{GHj}} = \lambda}$$



# MODELING OF TURBINE :-

## Block diagram Representation of steam turbines :-

The two common steam turbine system configurations

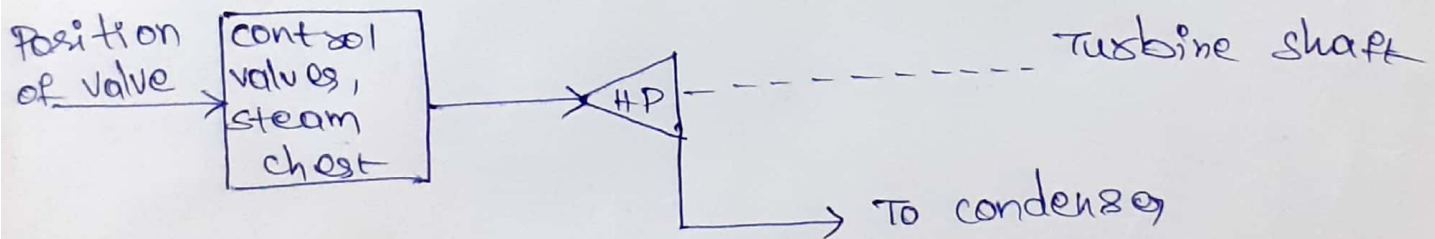
- are
- 1) Non-reheat type.
  - 2) Reheat type.

### 1) Non-reheat type :-

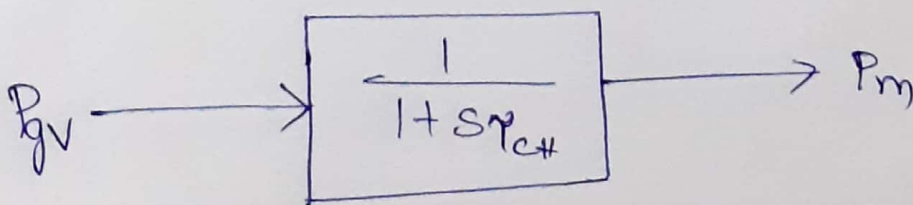
A simple non-reheat type turbine is modeled by a single time constant.

The functional block diagram representation of a non-reheat type of steam turbine is as shown in fig(i):

The approximate linear model of the non-reheat steam turbine is shown in fig(ii):-



fig(i) :- Block diagram representation of a non-reheat type of steam turbine.



fig(ii) :- Approximate linear model of a non-reheat steam turbine



Here,  $P_{GV}$  = Power at the gate of the Valve outlet (17)

$\tau_{CH}$  = the steam-chest time constant

$P_m$  = the mechanical power at the turbine shaft.

2) Reheat type :-

there are mainly two configurations and they are:

- i) Tandem compound system configuration
- ii) cross-compound system "

these two configurations are further classified into the following types:

- a) Tandem compound, single reheat type
- b) tandem compound, double reheat type.
- c) cross-compound, single reheat type with two low-pressure (LP) turbines
- d) cross-compound, single reheat type with single LP turbine
- e) cross-compound, double reheat type.

A tandem compound system has only one shaft on which all the turbines are mounted. The turbines

- are of
- a) High pressure (HP)
  - b) Low pressure (LP)
  - c) Intermediate pressure (IP) turbines
  - d) Very high pressure (VHP) "

1) tandem compound single reheat system :-

All compound steam turbines use governors - controlled valves, at the inlet to the HP turbine, to control the steam flow.

The steam chest, reheater, and cross-over piping introduce delays. these time delays are represented by

$\tau_{ch}$  = steam-chest time constant (from 0.1 to 0.4s)

$\tau_{RH}$  = Reheat time constant (from 4 to 11s)

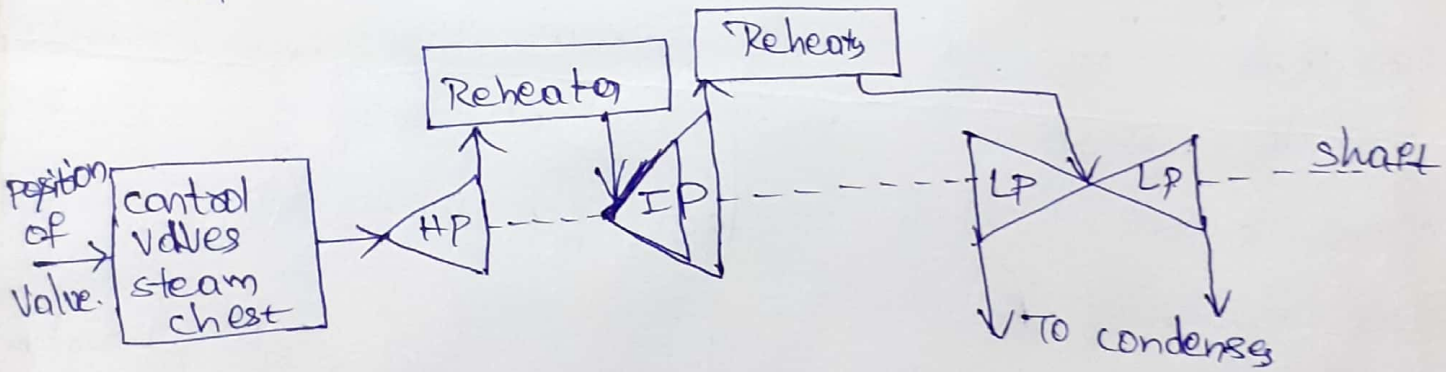
$\tau_{CO}$  = cross-over time constant (from 0.3 to 0.5s)

The fractions of total turbine power are represented by

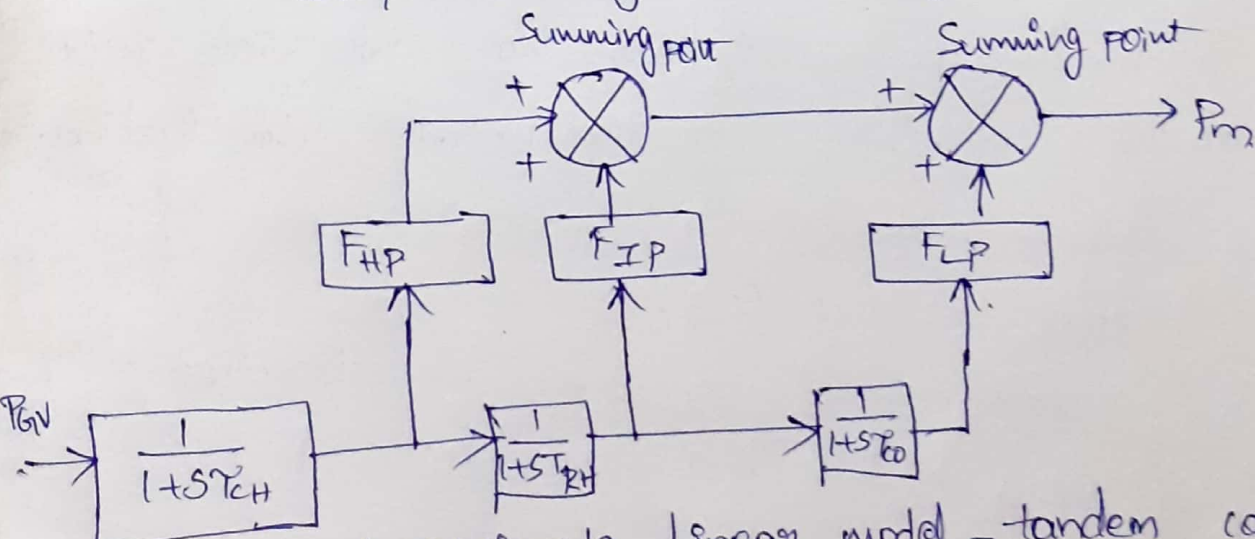
$F_{HP}$  = fraction of HP turbine power (typical value is 0.3)

$F_{IP}$  = fraction IP turbine power (typical value is 0.3)

$F_{LP}$  = fraction of LP turbine power (typical value is 0.4)



fig(a) :- Functional block diagram representation - tandem compound single reheat system.



fig(b) :- Approximate linear model - tandem compound single reheat system.



2) tandem compound double reheat system:-

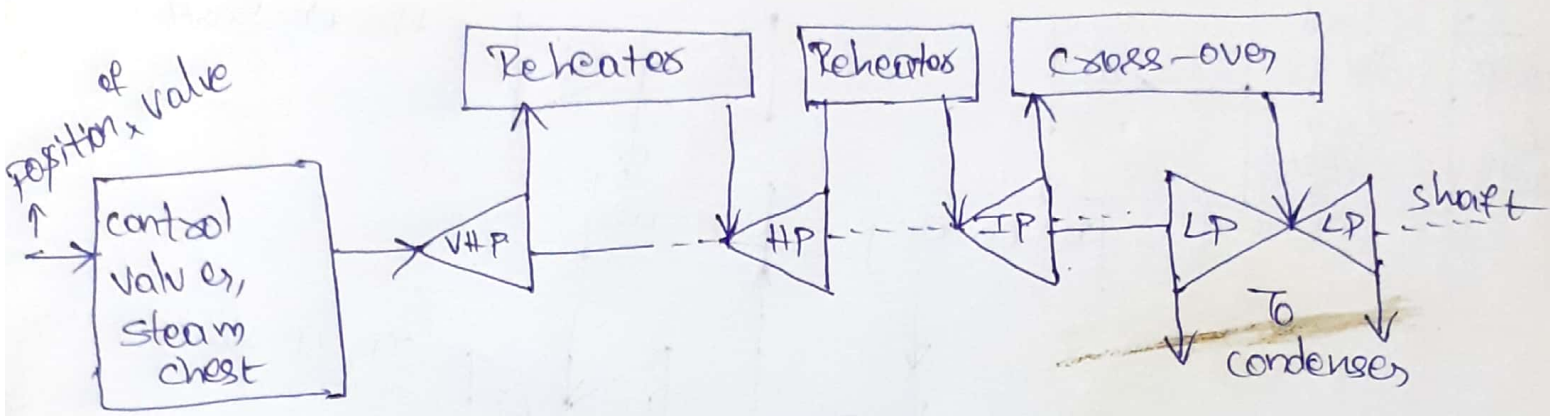


Fig (iii) :- functional block diagram representation - tandem compound double reheat system.

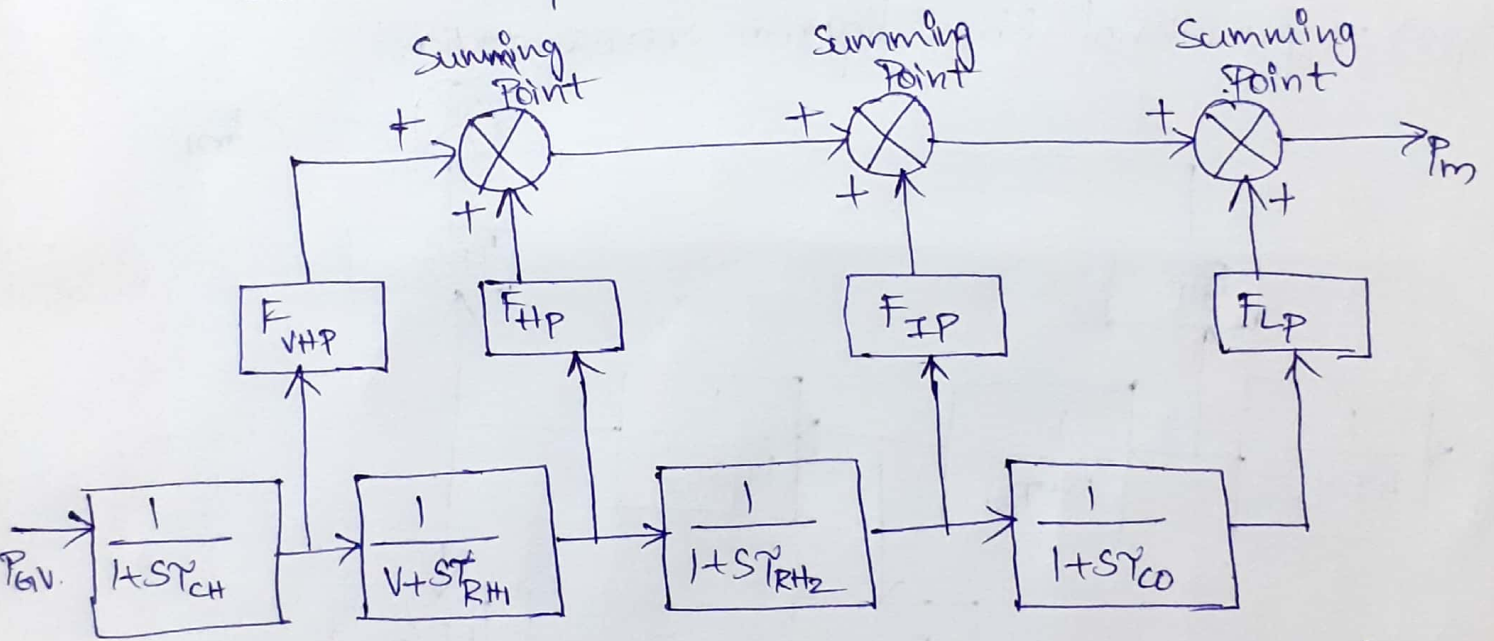


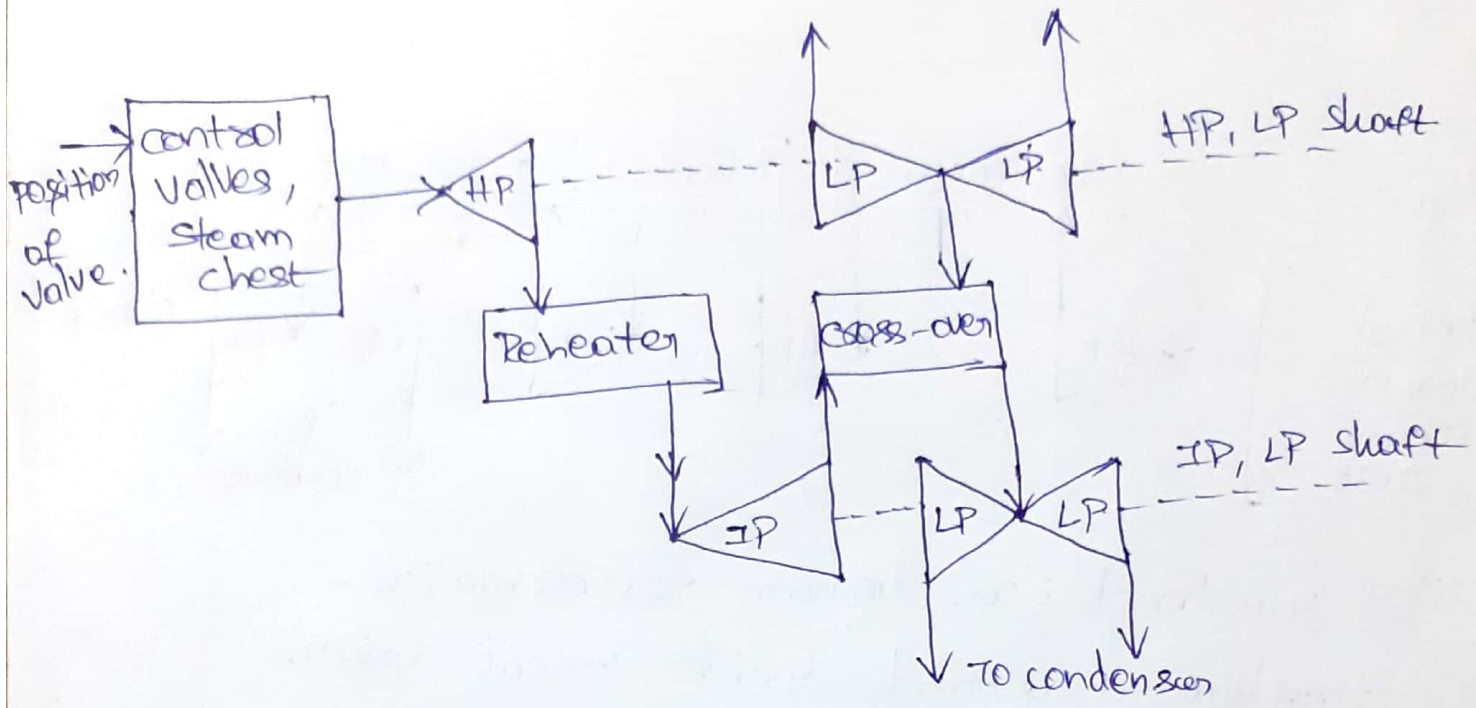
Fig (iv) :- Approximate linear model - tandem compound double reheat system.

The time delays are represented by:

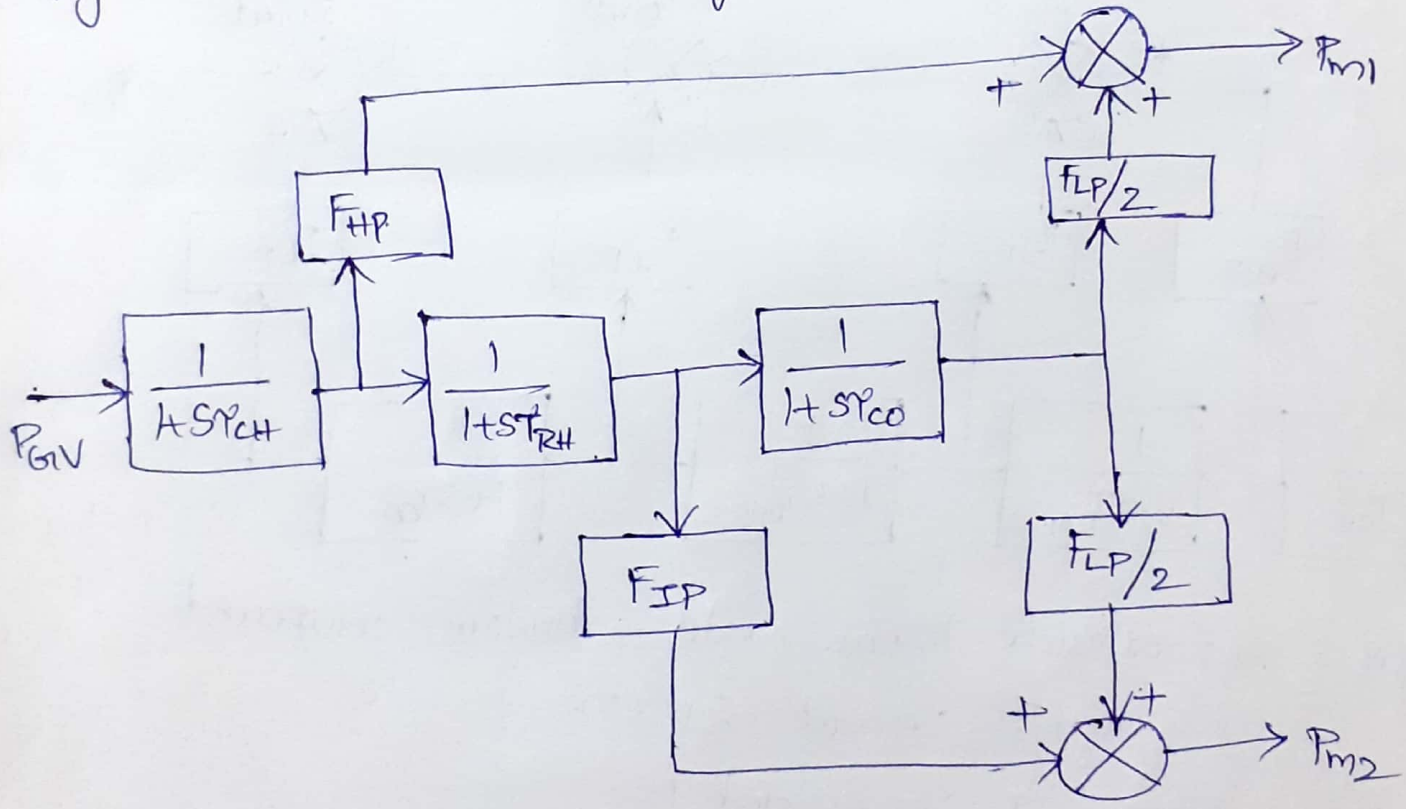
- $T_{RH1}$  = first reheat time constant
- $T_{RH2}$  = second reheat time constant.

3) cross-compound single reheat system (with two LP turbines):-

- the cross-compound single reheat system with two LP turbines is shown in fig (v) & fig (vi).



fig(v) :- Functional block diagram representation.



fig(vi) :- Approximate linear model. cross-compound single reheat system (two LP turbines)



4) Cross-compound single reheat system (with single LP turbine) (21)

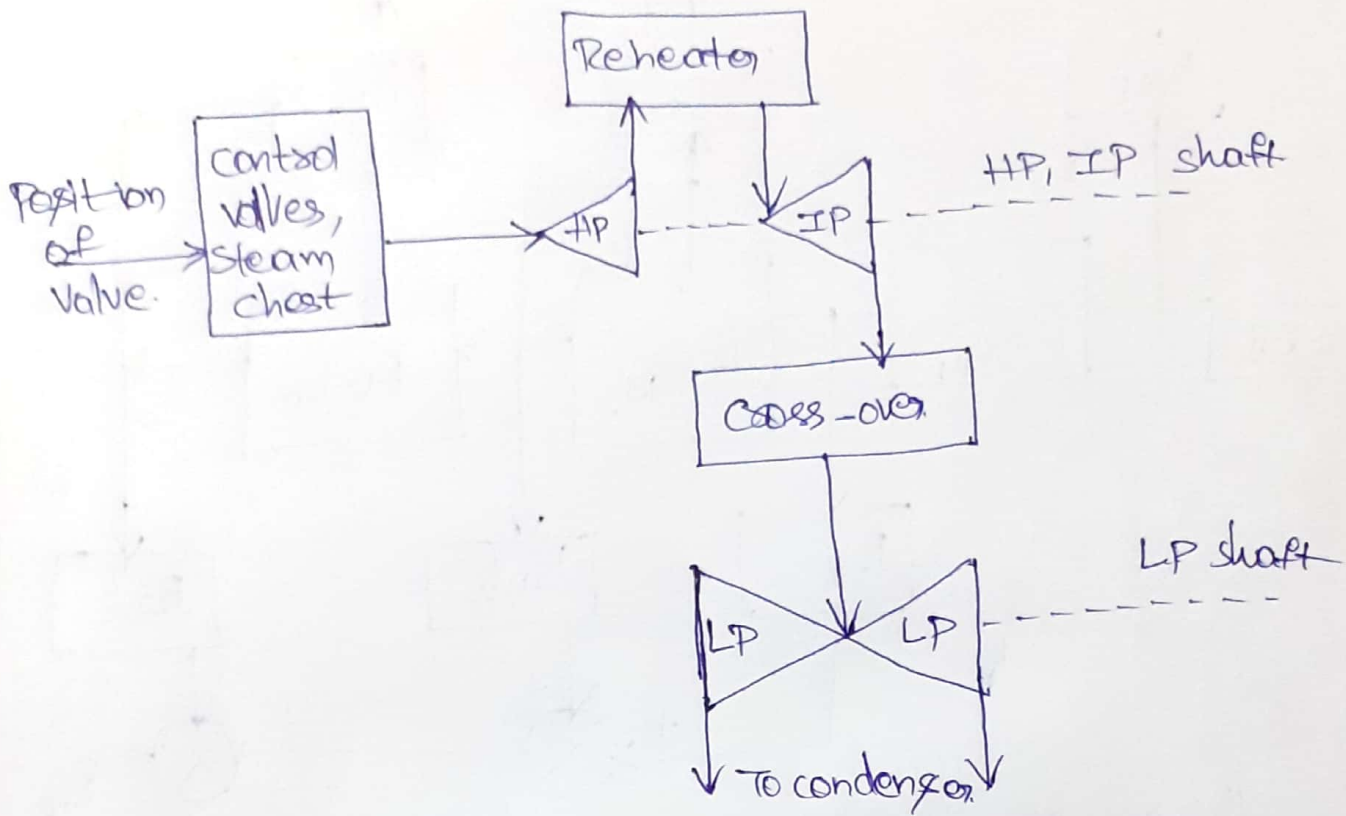


Fig. (vii) :- Functional block diagram representation

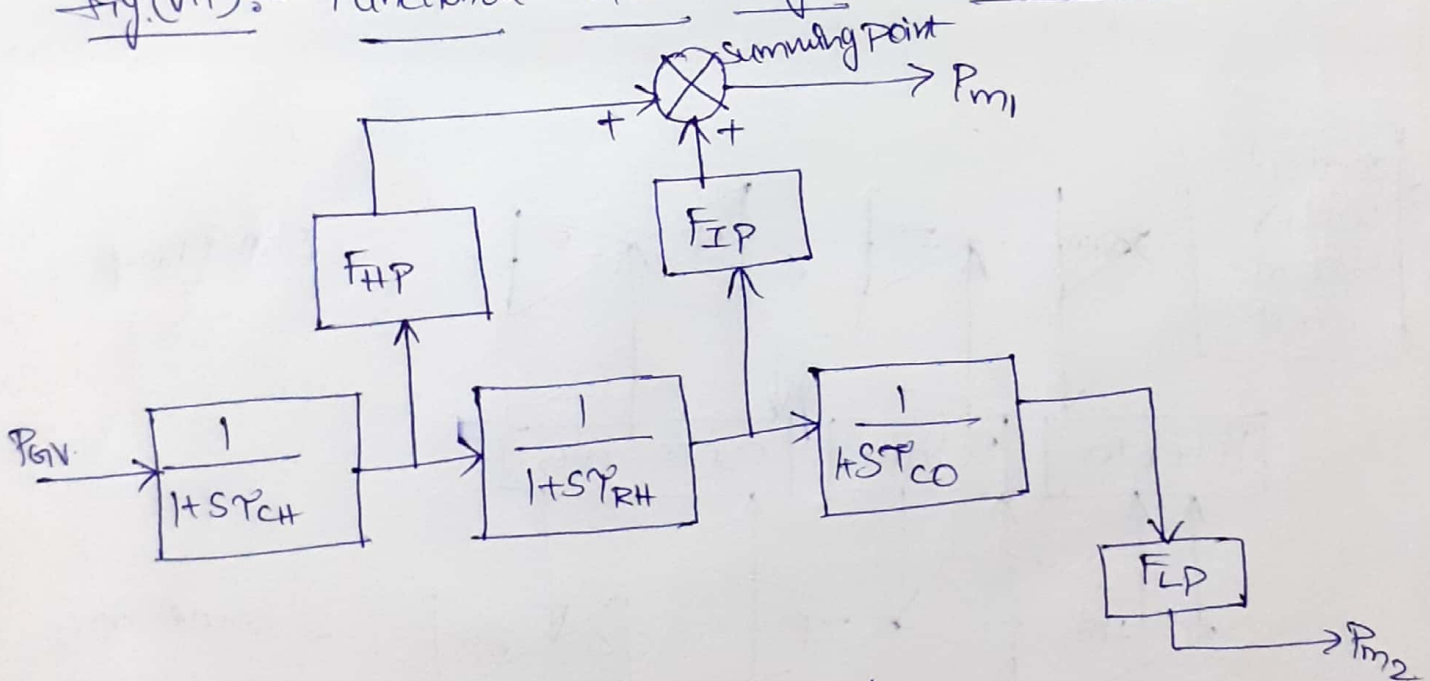
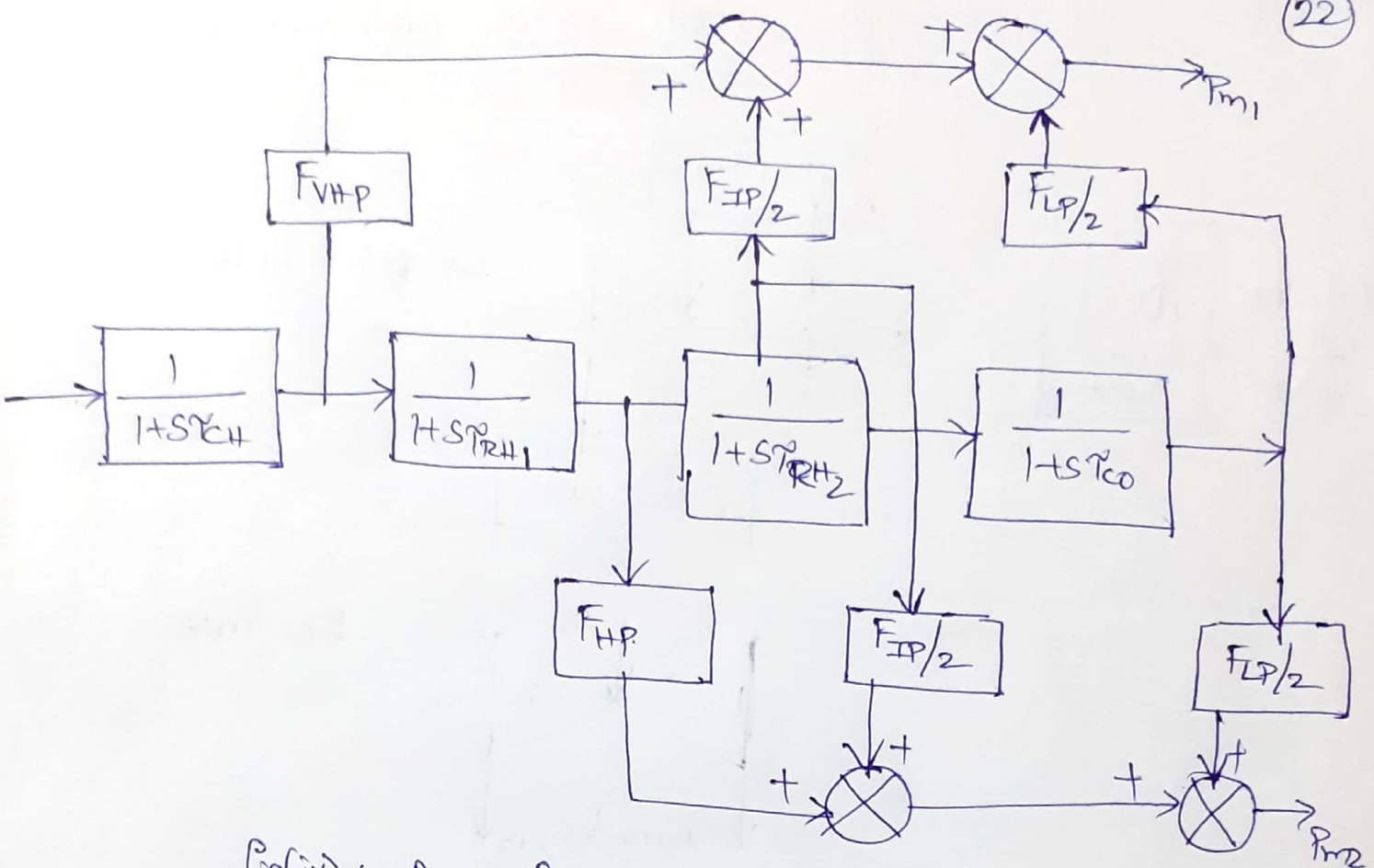


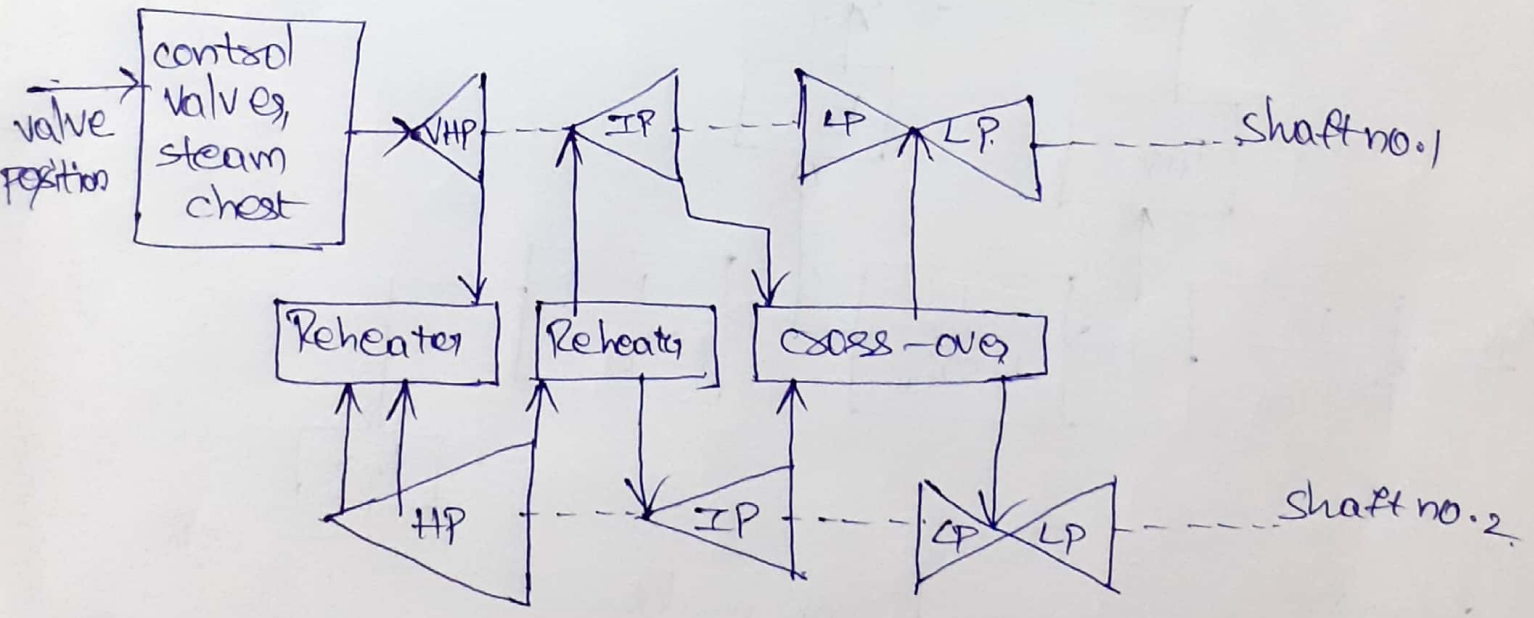
Fig. (viii) :- Approximate linear model.

5) Cross-compound double reheat type :-

the cross compound double reheat-type system is shown in fig (ix) & fig (x).



fig(ix) :- Approximate linear model.



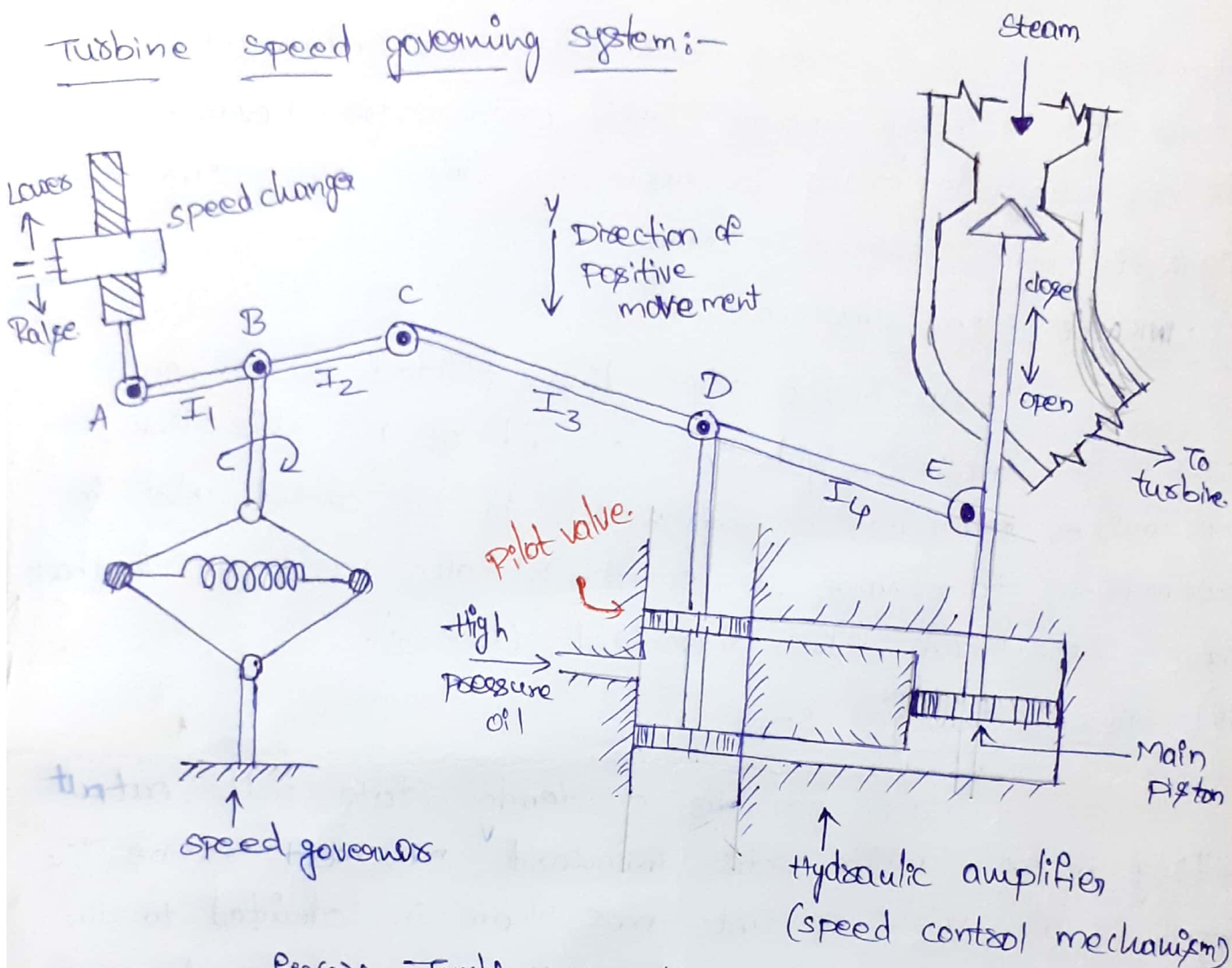
fig(x) :- Functional block diagram representation.



# MODELING OF GOVERNOR :-

Q23

## Turbine speed governing system :-



Fig(a) :- Turbine speed governing system

Fig(a) shows schematically the speed governing system of a steam turbine. The system consists of the following components:

### 1) FLY BALL SPEED GOVERNOR :-

This is the heart of the system which senses the change in speed (frequency). As the speed increases the fly balls move outwards and the point 'B' on linkage mechanism moves downwards. The reverse happens when the speed decreases.

ii) HYDRAULIC AMPLIFIER :-

It comprises a pilot valve and main piston arrangement. Low power level pilot valve movement is converted into high power level piston valve movement. This is necessary in order to open or close the steam valve against high pressure steam.

iii) LINKAGE MECHANISM :-

ABC is a rigid link pivoted at 'B' and 'CDE' is another rigid link pivoted at 'D'. This link mechanism provides a movement to the control valve in proportion to change in speed. It also provides a feedback from the steam valve movement (link-4).

iv) Speed changer :-

It provides a steady state power output setting for the turbine. Its downward movement opens the upper pilot valve so that more steam is admitted to the turbine under steady conditions (hence more steady power output). The reverse happens for upwards movement of speed changer.

MODEL OF SPEED GOVERNING SYSTEM :-

Assume that the system is initially operating under steady conditions - the linkage mechanism stationary and pilot valve closed, steam valve opened by a definite magnitude, turbine running at constant speed with turbine power output balancing the generator load.



Let, the operating conditions be characterised by (25)

$f^0$  = system frequency (speed)

$P_G^0$  = generator output = turbine output  
(neglecting generator loss)

$y_E^0$  = steam valve setting.

To obtain a linear incremental model around these operating conditions.

Let the point 'A' on the linkage mechanism be moved downwards by a small amount  $\Delta y_A$ . It is a command which causes the turbine power output to change and can therefore be written as

$$\Delta y_A = k_c \cdot \Delta P_c \longrightarrow \text{eq ①}$$

where,  $\Delta P_c$  is the commanded increase in power.

the command signal  $\Delta P_c$  (i.e.,  $\Delta y_E$ ) sets into motion a sequence of events :-

i) the pilot valve moves upwards, high pressure oil flows on to the top of the main piston moving it downwards; the steam valve opening consequently increases, the turbine generator speed increases, i.e., the frequency goes up.

Let us model these events mathematically.

Two factors contribute to the movement of C:

i)  $\Delta y_A$  contributes  $-\left(\frac{l_2}{l_1}\right) \Delta y_A$  (or)  $-k_1 \Delta y_A$  (i.e. upwards) or

$$-k_1 k_c \Delta P_c.$$

ii) Increase in frequency  $\Delta f$  causes the fly balls to move outwards so that 'B' moves downwards by a proportional amount  $k_2' \Delta f$ . The consequent movement of 'c' with 'A' remaining fixed at  $\Delta y_A$

$$\text{is } \boxed{+ \left( \frac{l_1 + l_2}{l_1} \right) k_2' \Delta f = + k_2 \Delta f} \quad (\text{ie downwards})$$

the net movement of 'c' is therefore

$$\boxed{\Delta y_c = -k_1 k_c \Delta p_c + k_2 \Delta f} \quad \longrightarrow \text{eq (2)}$$

the movement of D,  $\Delta y_D$ , is the amount by which the pilot valve opens. It is contributed by  $\Delta y_c$  &  $\Delta y_E$  and can be written as

$$\boxed{\Delta y_D = \left[ \frac{l_4}{l_3 + l_4} \right] \Delta y_c + \left[ \frac{l_3}{l_3 + l_4} \right] \Delta y_E} \quad \longrightarrow \text{eq (3)}$$

$$\Rightarrow \Delta y_D = k_3 \Delta y_c + k_4 \Delta y_E \quad \longrightarrow \text{eq (3)}$$

The movement  $\Delta y_D$  depending upon its sign opens one of the ports of the pilot valve admitting high pressure oil into the cylinders these by moving the main piston and opening the steam valve by  $\Delta y_E$ .

certain justifiable simplifying assumptions, which can be made at this stage, are;

i) Inertial reaction forces of main piston and steam valve are negligible compared to the forces exerted on the piston by high pressure oil.



$$\Delta Y_E(s) = \frac{k_1 k_3 k_c \Delta P_c(s) - k_2 k_3 \Delta F(s)}{\left[ k_{L4} + \frac{s}{k_5} \right]}$$

$$= \left[ \frac{\Delta P_c(s)}{R} - \Delta F(s) \right] \times \left[ \frac{k_{sg}}{1 + T_{sg} s} \right]$$

$$= \frac{k_3 \cdot \left[ k_1 k_c \Delta P_c(s) - k_2 \cdot \Delta F(s) \right]}{k_{L4} \cdot \left[ 1 + \frac{s}{k_4 k_5} \right]}$$

$$= \frac{k_1 k_3 k_c \cdot \left[ \Delta P_c(s) - \frac{k_2}{k_1 k_c} \Delta F(s) \right]}{k_{L4} \left[ 1 + \frac{s}{k_4 k_5} \right]}$$

$$= \left( \frac{k_1 k_3 k_c}{k_{L4}} \right) \left[ \Delta P_c(s) - \frac{1}{\left( \frac{k_1 k_c}{k_2} \right)} \Delta F(s) \right]$$

$$\Delta Y_E(s) = \frac{\left( \frac{k_1 k_3 k_c}{k_{L4}} \right) \left[ \Delta P_c(s) - \frac{1}{\left( \frac{k_1 k_c}{k_2} \right)} \Delta F(s) \right]}{1 + s \left( \frac{1}{k_4 k_5} \right)}$$

$$\rightarrow \Delta Y_E(s) = \frac{k_{sg} \left[ \Delta P_c(s) - \frac{1}{R} \Delta F(s) \right]}{1 + s T_{sg}}$$

→ eq (8)

where,  $R = \frac{k_1 k_c}{k_2}$  = speed regulation of the governor

$k_{sg} = \frac{k_1 k_3 k_c}{k_{L4}}$  = gain of speed governor

$T_{sg} = \frac{1}{k_4 k_5}$  = time constant of speed governor.

ii) Because of (i) above, the rate of oil admitted to the cylinder is proportional to port opening  $\Delta y_D$ . (25)

The volume of oil admitted to the cylinder is thus proportional to the time integral of  $\Delta y_D$ . The movement  $\Delta y_E$  is obtained by dividing the oil volume by the area of the cross-section of the piston. Thus.

$$\Delta y_E = k_5 \int_0^t (-y_D) dt \quad \longrightarrow \text{eq (4)}$$

It can be verified from the schematic diagram that a +ve movement  $\Delta y_D$ , causes negative (upward) movement  $\Delta y_E$  accounting for the negative sign used in eq (4)

taking the Laplace transform of eq (2), eq (3) & eq (4), we get

$$\Delta y_C(s) = -k_1 k_c \Delta P_C(s) + k_2 \Delta F(s) \quad \longrightarrow \text{eq (5)}$$

$$\Delta y_D(s) = k_3 \Delta y_C(s) + k_4 \Delta y_E(s) \quad \longrightarrow \text{eq (6)}$$

$$\Delta y_E(s) = -k_5 \frac{1}{s} \Delta y_D(s) \quad \longrightarrow \text{eq (7)}$$

eliminating  $\Delta y_C(s)$  &  $\Delta y_D(s)$ , we can write

$$\Delta y_D(s) = k_3 [-k_1 k_c \Delta P_C(s) + k_2 \Delta F(s)] + k_4 \Delta y_E(s)$$

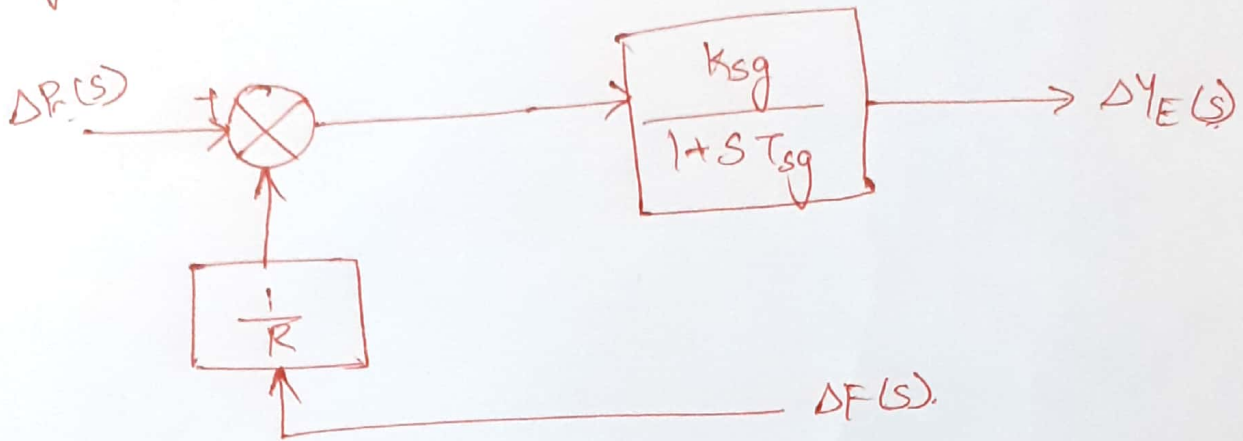
$$\Rightarrow \Delta y_E(s) = -\frac{k_5}{s} \left[ -k_1 k_3 k_c \Delta P_C(s) + k_2 k_3 \Delta F(s) + k_4 \Delta y_E(s) \right]$$

$$\Delta y_E(s) = \frac{k_1 k_3 k_5 k_c \Delta P_C(s) + k_2 k_3 k_5 \Delta F(s) + k_4 k_5 \Delta y_E(s)}{s}$$

$$\Rightarrow \Delta y_E(s) \left[ 1 + \frac{k_4 k_5}{s} \right] = \frac{k_1 k_3 k_5 k_c \Delta P_C(s) + k_2 k_3 k_5 \Delta F(s)}{s}$$



eq(8) is represented in the form of a block diagram in fig(b):-



Fig(b): Block diagram representation of speed governor system.

The speed governing system of a hydro-turbine is more involved. An additional f/b loop provides temporary droop compensation to prevent instability. This is necessitated by the large inertia of the penstock gate which regulates the rate of water input to the turbine.

LOAD FREQUENCY CONTROL

Necessity of keeping frequency constant :-

Introduction :-

In power system, both active & reactive power demands are never steady & they continually change with the rising or falling trend. Steam or water input to turbo-generators or hydro generators must therefore, be continuously regulated to match the active power demand, failing which the machine speed will vary with consequent change in frequency & it may be highly undesirable. The maximum permissible change in frequency is  $\pm 2\%$ .

Also, the excitation of the generators must be continuously regulated to match the reactive power demand with reactive power generation; otherwise, the voltages at various system buses may go beyond the prescribed limits. In modern large interconnected systems, manual regulation is not feasible & therefore automatic generation & voltage regulation equipment is installed on each generator. The controllers are set for a particular operating condition & they take care of small changes in load demand without exceeding the limits of frequency & voltage. As the change in load demand becomes large, the controllers must be reset either manually or automatically.



(2)

Necessity of Maintaining Frequency constant :-  
constant frequency is to be maintained for the following functions:

- 1) All the AC motors should require constant frequency supply so as to maintain speed constant.
- 2) In continuous process industry, it affects the operation of the process itself.
- 3) For synchronous operation of various units in the power system network, it is necessary to maintain frequency constant.
- 4) Frequency affects the amount of power transmitted through interconnecting lines.
- 5) Electrical clocks will lose or gain time if they are driven by synchronous motors, & the accuracy of the clocks depends on frequency & also the integral of this frequency error is loss or gain of time by electric clocks.

CONTROL AREA :- concept :-

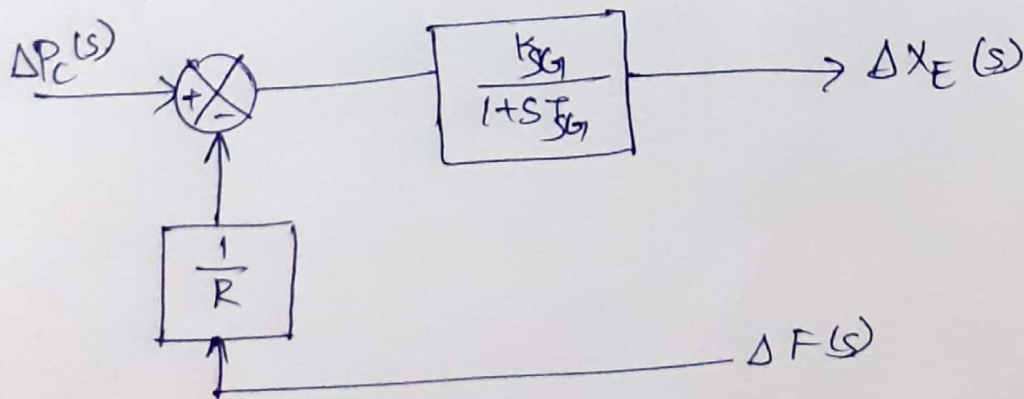
In a practical power system consisting of number of generating stations & loads. "the possibility of the division of an extended power system (say, national grid) into subareas (may be, state electricity boards) in which the generators are tightly coupled together so as to

form a coherent group, i.e. all the generators respond <sup>(3)</sup> in unison to changes in load or speed changes settings. Such a coherent area is called a "control area" in which the frequency is assumed to be the same through out in static as well as dynamic conditions.

For purposes of developing a suitable control strategy, a control area can be reduced to a single speed governor, turbo-generator and load system. All the control strategies discussed so far are, therefore, applicable to an independent control area.

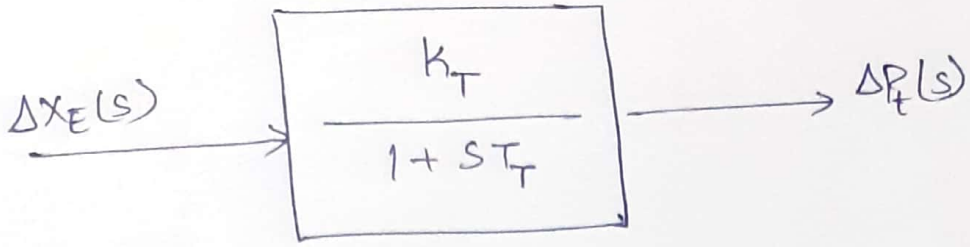
### SINGLE AREA CONTROL :-

A complete block diagram representation of an isolated power system comprising turbine, generator, governor and load is easily obtained by combining the block diagrams of individual components i.e. turbine, speed governing system, generator load model.. i.e

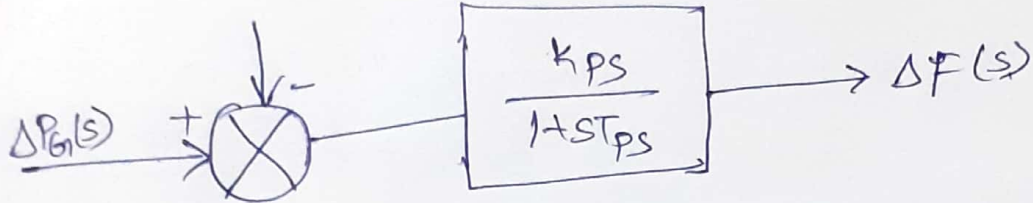


fig(a) :- Block diagram representation of speed governing for steam turbine.

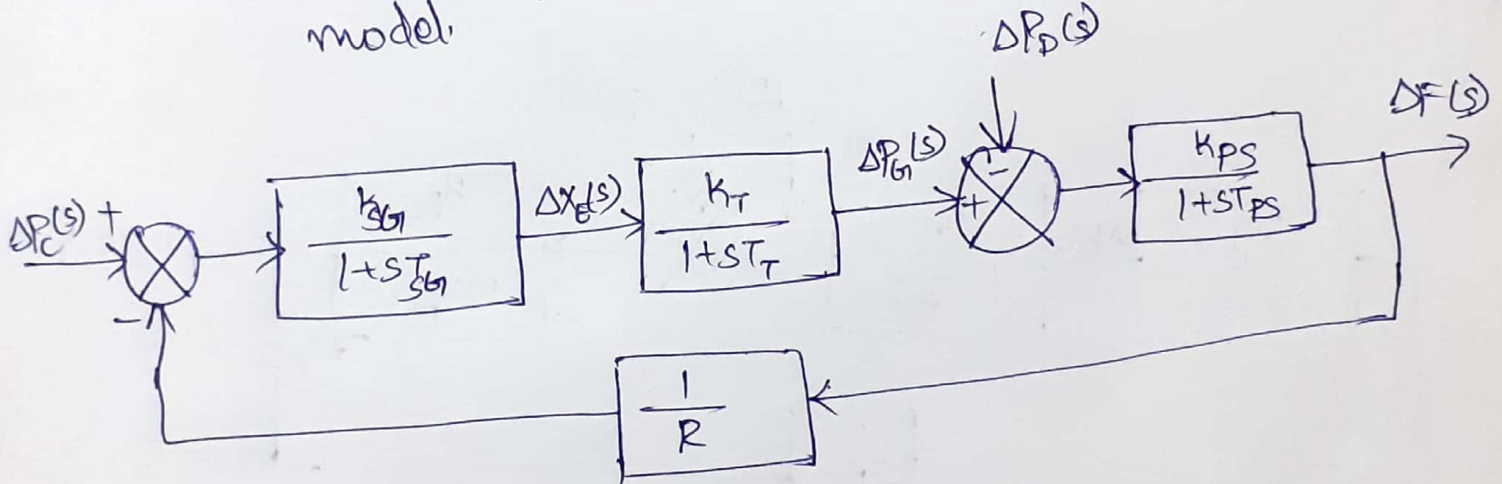




fig(b):- turbine transfer function model.



fig(c):- Block diagram representation of generator-load model.



where  $\Delta P_G(s) = \Delta P_L(s)$

fig(d):- Block diagram representation of an isolated power system

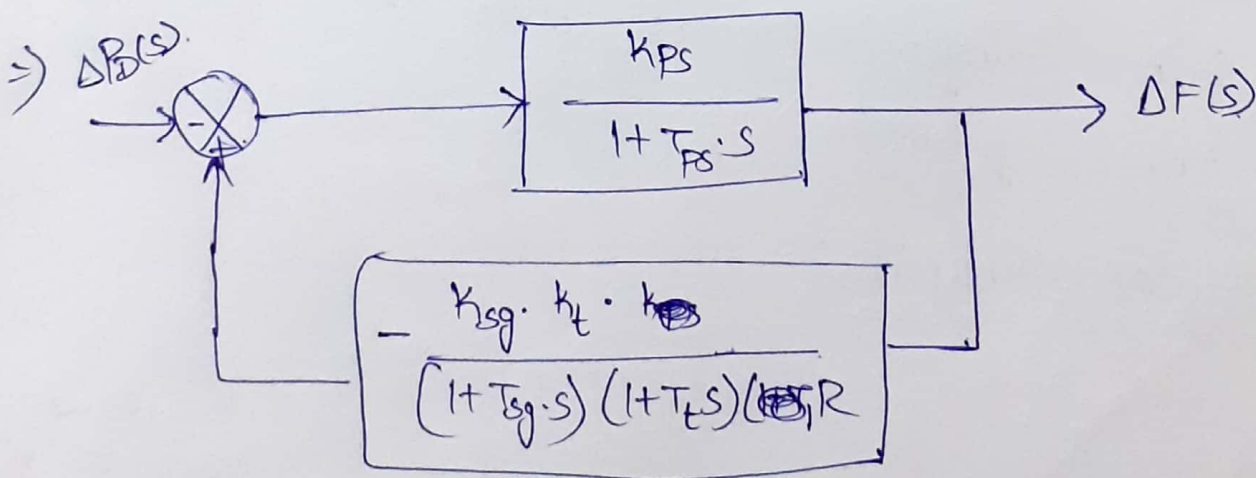
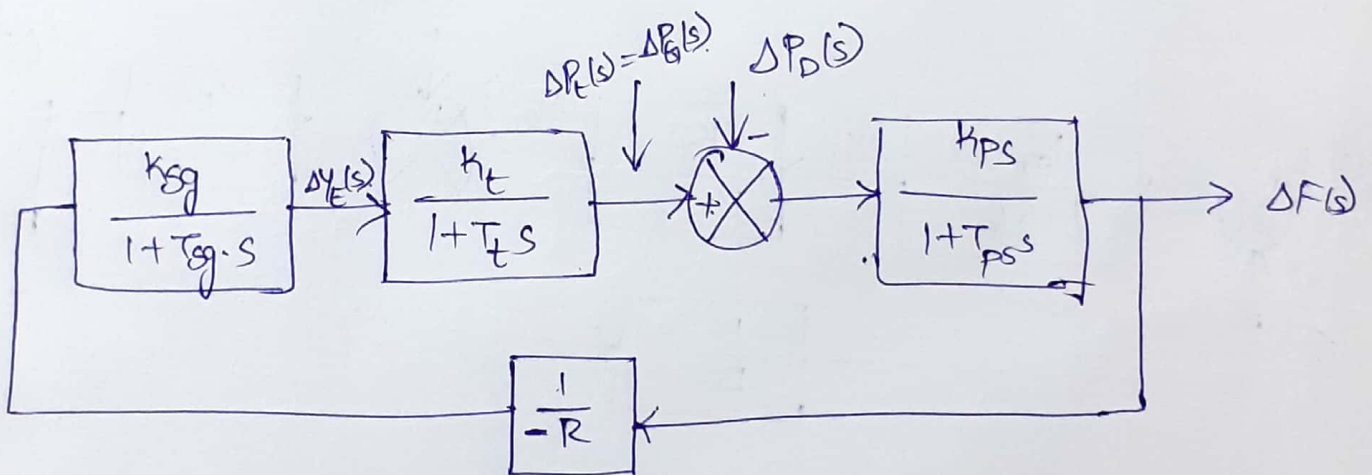
Steady state Analysis :- (UNCONTROLLED CASE):-

The model of fig(d). shows that there are two important incremental inputs to the load frequency control system -  $\Delta P_C$ , the change in speed changes setting; and  $\Delta P_D$ , the change in load demand.

Let us consider a simple situation in which the speed  $\omega$  changes has a fixed setting (i.e.  $\Delta P_c = 0$ ) and the load demand changes. This is known as "free governor operation".

For such an operation the steady change in system frequency for a sudden change in load demand by an amount  $\Delta P_D$  (i.e.,  $\Delta P_D(s) = \frac{\Delta P_D}{s}$ ) is obtained as follows:

We know that, for such an operation, the steady-state change of frequency  $\Delta f$  is to be estimated from the block diagram of fig(d), when  $\Delta P_c(s) = 0$  the fig(d) becomes.





$$\frac{\Delta F(s)}{-\Delta P_D(s)} = \frac{\left( \frac{k_{ps}}{1+sT_{ps}} \right)}{\left[ 1 + \frac{k_{sg} k_t k_{ps}}{(1+sT_{sg})(1+sT_t)(1+sT_{ps})R} \right]}$$

$$\Delta F(s) = \frac{k_{ps}}{(1+sT_{ps}) + \frac{k_{sg} \cdot k_t}{(1+sT_{sg})(1+sT_t)R}} \times (-\Delta P_D(s))$$

$$\Rightarrow \Delta F(s) = \frac{-k_{ps}}{(1+sT_{ps}) + \frac{(k_{sg} \cdot k_t / R)}{(1+sT_{sg})(1+sT_t)}} \times \frac{\Delta P_D}{s}$$

→ eq(1)

We know that

$$\Delta P \Big|_{\substack{\text{steady state} \\ \Delta P_c = 0}} = \lim_{s \rightarrow 0} s \cdot \Delta F(s) \Big|_{\Delta P_c(s) = 0}$$

$$\Delta P \Big|_{\substack{\text{steady state} \\ \Delta P_c = 0}} = \lim_{s \rightarrow 0} \frac{-k_{ps}}{(1+sT_{ps}) + \frac{(k_{sg} \cdot k_t / R)}{(1+sT_{sg})(1+sT_t)}} \cdot \frac{\Delta P_D}{s}$$

$$\Rightarrow \left. \begin{array}{l} \Delta f \\ \text{steady state} \\ \Delta P_c = 0 \end{array} \right\} = - \left[ \frac{k_{ps}}{1 + (k_{sg} k_t k_{ps}/R)} \right] \Delta P_D$$

→ eq(2)

while the gain 'k<sub>t</sub>' is fixed for the turbine

k<sub>ps</sub> is fixed for power system,

k<sub>sg</sub>, the speed governor gain is easily adjustable by changing lengths of various links

Let it be assumed for simplicity, that k<sub>sg</sub> is so adjusted that

$$k_{sg} k_t \approx 1$$

it is also recognized that  $k_{ps} = \frac{1}{B}$ , where

$$B = \frac{\Delta P_D}{\Delta f} / P_0 \quad (\text{in pu MW/unit change in frequency})$$

Now, 
$$\Delta f = - \left[ \frac{\left(\frac{1}{B}\right)}{1 + \frac{1 \cdot 1}{BR}} \right] \Delta P_D$$

$$\Rightarrow \Delta f = - \left[ \frac{1}{\left(B + \frac{1}{R}\right)} \right] \Delta P_D \quad \rightarrow \text{eq(3)}$$

The above equation gives the steady state changes in frequency caused by changes in load demand. speed regulation R is naturally so adjusted that changes in frequency are small.

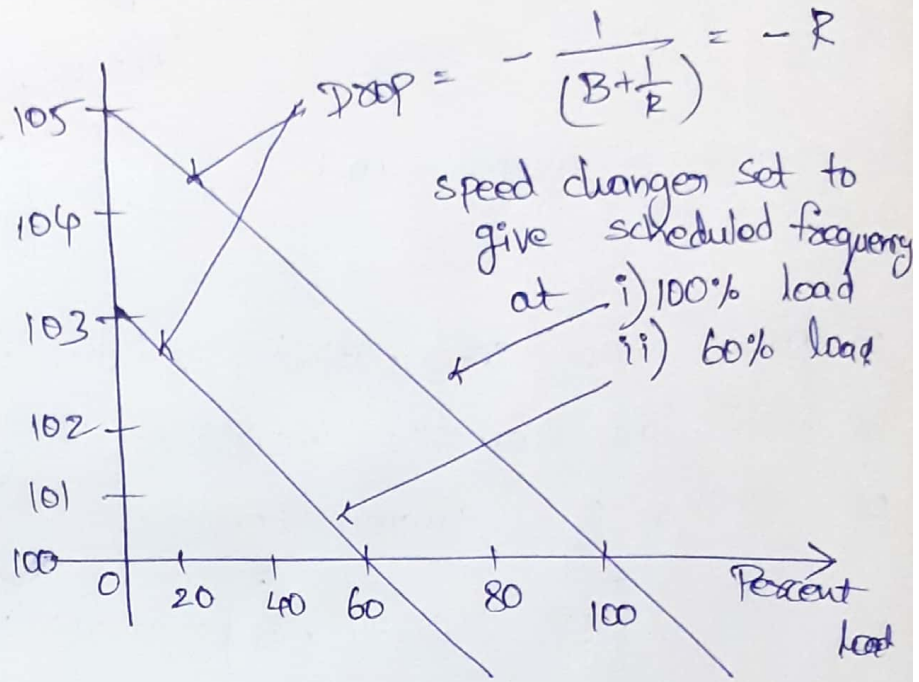


(of the order of 5% from no load to full load).

9

∴ There fore, the linear incremental relation eq(3) can be applied from no load to full load.

with this understanding fig(e) can be applied from no load to full load. Shows the linear relationship between frequency and load for free governor operation with speed changes set to give a scheduled frequency of 100% at full load.



fig(e) :- Steady state load - frequency characteristic of a speed governor system.

the 'droop' or slope of this relationship is  $-\left[\frac{1}{B + \frac{1}{R}}\right]$

Power system parameter  $B$  is generally much smaller than  $\frac{1}{R}$  (a typical value is  $B = 0.01$  pu MW/Hz and  $\frac{1}{R} = \frac{1}{3}$ ) so that  $B$  can be neglected in comparison. Eq(3) then simplifies to

$$\Delta P = -R(\Delta P_D) \longrightarrow \text{eq(4)}$$

The droop of the load frequency curve is thus mainly determined by  $R$ , the speed governor regulation.

It is also observed from the above that increase in load demand ( $\Delta P_D$ ) is met under steady conditions partly by increased generation ( $\Delta P_G$ ) due to opening of the steam valve and partly

by decreased load demand due to drop in system frequency. (10)

From the block diagram of fig(d). (with  $k_g k_t \approx 1$ )

$$\Delta P_G = -\frac{1}{R} \Delta f = \left[ \frac{1}{1+BR} \right] \Delta P_D$$

$\Delta f = -2 \Delta B$   
as  $\Delta B \approx \Delta P_G$

Decrease in system load =  $B \Delta f = \left[ \frac{BR}{BR+1} \right] \Delta P_D$

$\Delta f = -2 \Delta P_G$   
 $\Delta P_G = -\frac{1}{2} \Delta f$

of course, the contribution of decrease in system load is much less than the increase in generation. For typical values of 'B' & 'R' quoted earlier.

$\Delta P_G = 0.971 \Delta P_D$

→ (5)

$$\Delta P_G = \frac{1}{R} \left[ \frac{-1}{B + \frac{1}{R}} \right] \Delta P_D$$

(from eq 5)

Decrease in system load =  $0.029 \Delta P_D$       $\Delta P_G = \frac{1}{R} \left[ \frac{-R}{BR+1} \right]$

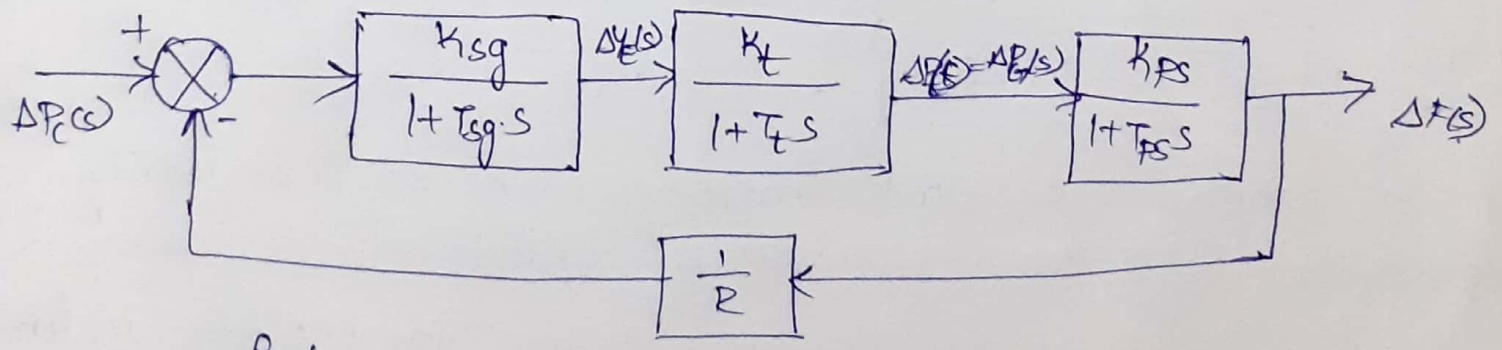
CONTROLLED CASE :- (Load demand is constant,  $\Delta P_D = 0$ )

considers a step change in a speed-changes position with the load demand remaining fixed i.e.,

$\Delta P_c(s) = \frac{\Delta P_c}{s}$  and  $\Delta P_D = 0$

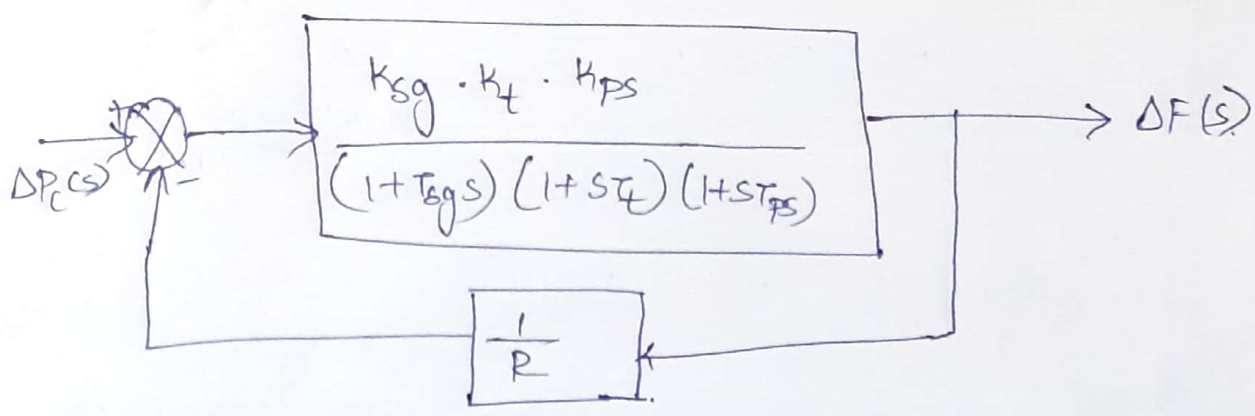
→ eq(6)

when  $\Delta P_D = 0$ , fig(d) becomes



fig(1)





$$\Rightarrow \frac{\Delta F(s)}{\Delta P_c(s)} = \frac{\left[ \frac{K_{sg} \cdot K_t \cdot K_{ps}}{(1 + sT_{sg})(1 + sT_t)(1 + sT_{ps})} \right]}{1 + \frac{K_{sg} \cdot K_t \cdot K_{ps}}{(1 + sT_{sg})(1 + sT_t)(1 + sT_{ps})} R}$$

$$\Rightarrow \Delta F(s) = \frac{K_{sg} \cdot K_t \cdot K_{ps}}{(1 + sT_{sg})(1 + sT_t)(1 + sT_{ps}) + \left( \frac{K_{sg} \cdot K_t \cdot K_{ps}}{R} \right)} \cdot \left( \frac{\Delta P_c}{s} \right) \rightarrow \text{eq(7)}$$

We know that,

$$\Delta F \Big|_{\substack{\text{steady state} \\ \Delta P_D = 0}} = \lim_{s \rightarrow 0} s \Delta F(s) \Big|_{\Delta P_D(s) = 0}$$

$$\Rightarrow \Delta F \Big|_{\text{stead state}} = \left[ \frac{K_{sg} \cdot K_t \cdot K_{ps}}{1 + \frac{K_{sg} \cdot K_t \cdot K_{ps}}{R}} \right] \Delta P_c \rightarrow \text{eq(8)}$$

IF  $K_{sg} K_t \approx 1$

$$\Rightarrow \Delta F = \left[ \frac{K_{ps}}{1 + \frac{K_{ps}}{R}} \right] \Delta P_c = \left[ \frac{K_{ps}}{K_{ps} \left( \frac{1}{K_{ps}} + \frac{1}{R} \right)} \right] \Delta P_c = \frac{1}{B + \frac{1}{R}} \times \Delta P_c$$

$$\therefore \Delta f = \frac{1}{\left(B + \frac{1}{R}\right)} \Delta P_c \quad \left\{ \because k_{ps} = \frac{1}{B} \Rightarrow \frac{1}{k_{ps}} = B \right\} \quad (8)$$

$$\Rightarrow \Delta f = \left[ \frac{1}{B + \frac{1}{R}} \right] \Delta P_c \quad \rightarrow \text{eq (9)}$$

If the speed changer setting is changed by  $\Delta P_c$  while the load demand changes by  $\Delta P_D$ , the steady frequency change is obtained by superposition i.e.

$$\Delta f = \text{eq (3)} + \text{eq (9)}$$

$$\Rightarrow \Delta f = - \frac{1}{\left(B + \frac{1}{R}\right)} \Delta P_D + \frac{1}{\left(B + \frac{1}{R}\right)} \Delta P_c$$

$$\Delta f = \frac{1}{\left(B + \frac{1}{R}\right)} \left[ \Delta P_c - \Delta P_D \right] \quad \rightarrow \text{eq (10)}$$

According to eq (10) the frequency change caused by load demand can be compensated by changing the setting of the speed changer i.e.,

$$\Delta P_c = \Delta P_D \quad \text{for } \Delta f = 0$$

fig(e), depicts two load-frequency plots - one to give scheduled frequency at 100% rated load and the other to give the same frequency at 60% rated load.



## DYNAMIC RESPONSE :- of (Single Area control) (13)

Dynamic response means how the frequency changes as a function of time immediately after disturbance before it reaches the new steady-state condition.

The analysis of dynamic response requires the solution of dynamic equation of the system for a given disturbance.

The solution involves the solution of differential equations representing the dynamic behavior of the system.

The characteristic equation can be approximated as first order by examining the relative magnitudes of the time constants involved.

The inverse Laplace transform of  $\Delta F(s)$  gives the variation of frequency with respect to time for a given step change in load demand.

For a practical LFC system,

$$T_{sg} < T_t \ll T_{ps}$$

Typical values are:  $T_{sg} = 0.4s$ ,  $T_t = 0.5s$ ,  $T_{ps} = 20s$ .

If  $T_{sg}$  and  $T_t$  are considered negligible compared to  $T_{ps}$  and by adjusting  $k_{sg}k_t = 1$ , the block diagram of LFC of the power system of an isolated system is reduced to a first-order system as shown in fig.) the change in frequency is given by

$$\Delta F(s) \Big|_{\Delta P_c(s)=0} = \left[ \frac{\left( \frac{k_{ps}}{1+sT_{ps}} \right)}{\left( 1 + \frac{k_{ps}}{1+sT_{ps}} \times \frac{1}{R} \right)} \right] * \frac{-\Delta P_D}{s}$$

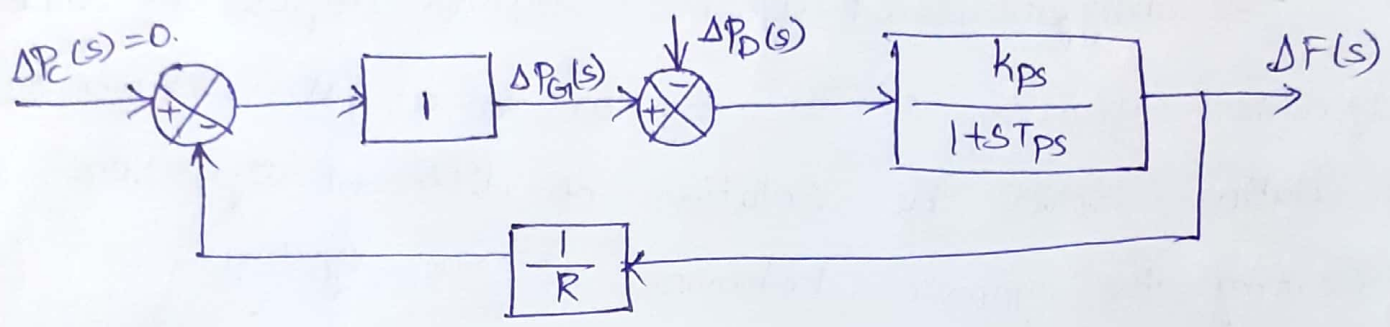


fig ( ) :- First order approximate block diagram of LFC of an isolated area.

$$\Rightarrow \Delta F(s) \Big|_{\Delta P_c(s)=0} = \left[ \frac{\left( \frac{k_{ps}}{1+sT_{ps}} \right)}{\left( (1+sT_{ps}) + \frac{k_{ps}}{R} \right)} \right] * \frac{-\Delta P_D}{s}$$

$$= \frac{-k_{ps}}{1+sT_{ps} + \frac{k_{ps}}{R}} * \frac{\Delta P_D}{s}$$

$$= \frac{-k_{ps}}{sT_{ps} + \left( 1 + \frac{k_{ps}}{R} \right)} * \frac{\Delta P_D}{s}$$

$$= \frac{-k_{ps}}{sT_{ps} + \left( \frac{R+k_{ps}}{R} \right)} * \frac{\Delta P_D}{s}$$



$$\Rightarrow \Delta F(s) \Big|_{\Delta P_c(s)=0} = \frac{-k_p s}{T_{ps} \left[ s + \frac{R+k_p s}{R \cdot T_{ps}} \right]} * \frac{\Delta P_D}{s}$$

$$= \frac{-\left( \frac{k_p s}{T_{ps}} \right)}{s + \left( \frac{R+k_p s}{R \cdot T_{ps}} \right)} * \frac{\Delta P_D}{s}$$

$$= \frac{-k_p s \cdot \Delta P_D}{T_{ps}} \left[ \frac{1}{s \left( s + \frac{R+k_p s}{R \cdot T_{ps}} \right)} \right]$$

Let  $\frac{R+k_p s}{R \cdot T_{ps}} = a \rightarrow \text{eq(1)}$

$$\Rightarrow \frac{1}{s \left( s + \frac{R+k_p s}{R \cdot T_{ps}} \right)} = \frac{1}{s(s+a)} = \frac{A}{s} + \frac{B}{s+a} \rightarrow \text{eq(2)}$$

$$\Rightarrow 1 = A(s+a) + Bs$$

put  $s = -a \Rightarrow 1 = A(-a+a) + B(-a)$

$$1 = -Ba$$

$$\Rightarrow \boxed{B = -\frac{1}{a}}$$

put  $s = 0$

$$\Rightarrow 1 = A(a) + 0 \Rightarrow \boxed{A = \frac{1}{a}}$$

Sub 'A' & 'B' values in eq(2)

$$\Rightarrow \frac{1}{s \left( s + \frac{R+k_p s}{R \cdot T_{ps}} \right)} = \frac{\left( \frac{1}{a} \right)}{s} + \frac{\left( -\frac{1}{a} \right)}{s+a}$$

$$\Rightarrow \text{W.K.T } a = \frac{R+kps}{R \cdot Tps}$$

$$\Rightarrow \frac{1}{s \left( s + \frac{R+kps}{R \cdot Tps} \right)} = \frac{\left( \frac{R \cdot Tps}{R+kps} \right)}{s} - \frac{\left( \frac{R \cdot Tps}{R+kps} \right)}{s + \left( \frac{R+kps}{R \cdot Tps} \right)}$$

⇒ eq (1) becomes

$$\left. \frac{\Delta F(s)}{\Delta P_c(s)=0} \right| = \frac{-kps \cdot \Delta P_D}{\bullet \cdot Tps} \cdot \left[ \frac{\left( \frac{R \cdot Tps}{R+kps} \right)}{s} - \frac{\left( \frac{R \cdot Tps}{R+kps} \right)}{s + \left( \frac{R+kps}{R \cdot Tps} \right)} \right]$$

$$= \frac{-kps \cdot \Delta P_D}{\bullet \cdot Tps} \cdot \frac{R \cdot Tps}{(R+kps)} \left[ \frac{1}{s} - \frac{1}{s + \left( \frac{R+kps}{R \cdot Tps} \right)} \right]$$

$$\left. \frac{\Delta F(s)}{\Delta P_c(s)=0} \right| = \frac{-kps \cdot R \cdot \Delta P_D}{R+kps} \left[ \frac{1}{s} - \frac{1}{s + \left( \frac{R+kps}{R \cdot Tps} \right)} \right]$$

W.K.T  $\Delta F(t) = \mathcal{L}^{-1} \Delta F(s)$

$$\Delta F(t) = \frac{-kps \cdot R \cdot \Delta P_D}{R+kps} \left[ 1 - e^{-t \left( \frac{R+kps}{R \cdot Tps} \right)} \right]$$

$$\Delta F(t) = \frac{-kps \cdot R}{R+kps} \left[ 1 - e^{-\frac{t}{Tps} \left( \frac{R+kps}{R} \right)} \right] \Delta P_D$$

$$\left. \begin{aligned} \therefore \mathcal{L}^{-1} \left( \frac{1}{s} \right) &= 1 \\ \mathcal{L}^{-1} \left( \frac{1}{s+a} \right) &= e^{-ta} \end{aligned} \right\}$$

→ eq (3)



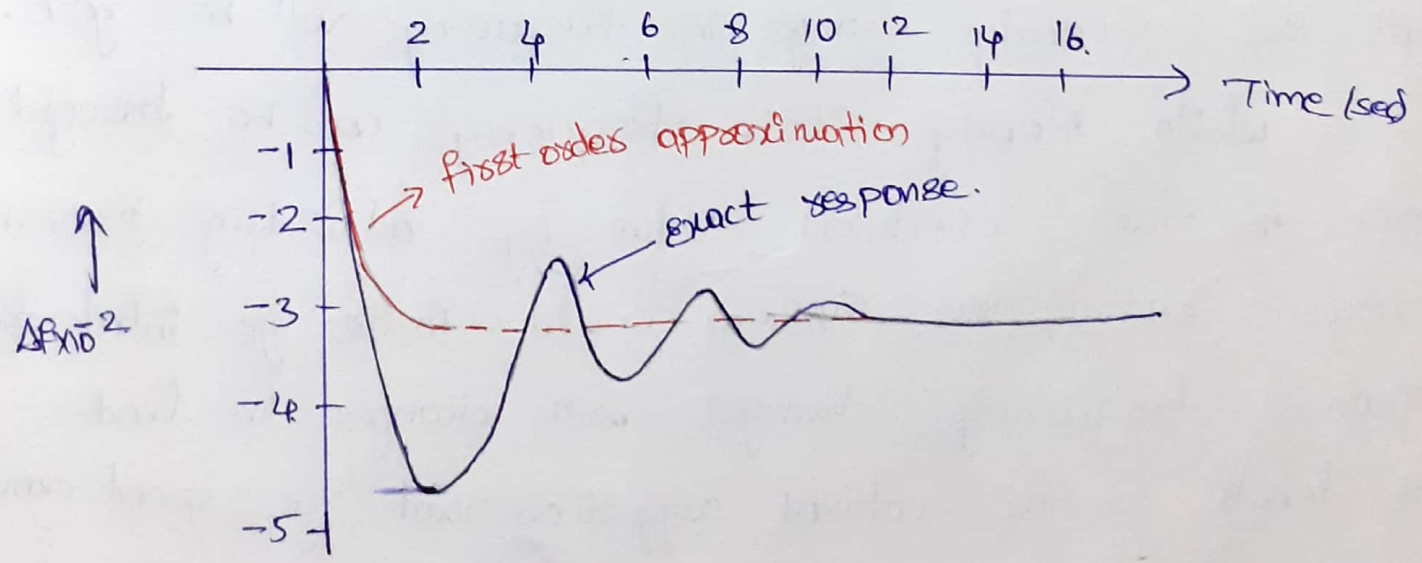
Taking  $R=3$ ,  $K_{PS} = \frac{1}{B} = 100$ ,  $T_{PS} = 20$ ,  $\Delta P_D = 0.01$  PU

$$\therefore \Delta f(t) = \frac{3 \times 100}{3 + 100} \left[ 1 - e^{-\frac{t}{20} \left( \frac{3}{3+100} \right)} \right] \times 0.01$$

$$\Delta f(t) = -0.029 \left[ 1 - e^{-1.777t} \right] \rightarrow \text{eq (4) a}$$

$$\Rightarrow \Delta f|_{\text{steady state}} = -0.029 \text{ Hz} \left[ \text{as } e^{-1.777t} \approx 0 \text{ for steady state.} \right] \rightarrow \text{eq (4) b}$$

The plot of change in frequency versus time for first order approximation and exact response are shown in fig(). First order approximation is obviously a poor approximation.



fig() :- Dynamic response of change in frequency for a step change in load. ( $\Delta P_D = 0.01$  PU,  $T_{sg} = 0.4$  sec,  $T_f = 0.5$  sec,  $T_{PS} = 20$  sec,  $K_{PS} = 100$ ,  $R = 3$ ).

## LOAD FREQUENCY CONTROL OF 2-AREA System :-

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### PROPORTIONAL PLUS INTEGRAL CONTROL OF SINGLE AREA :-

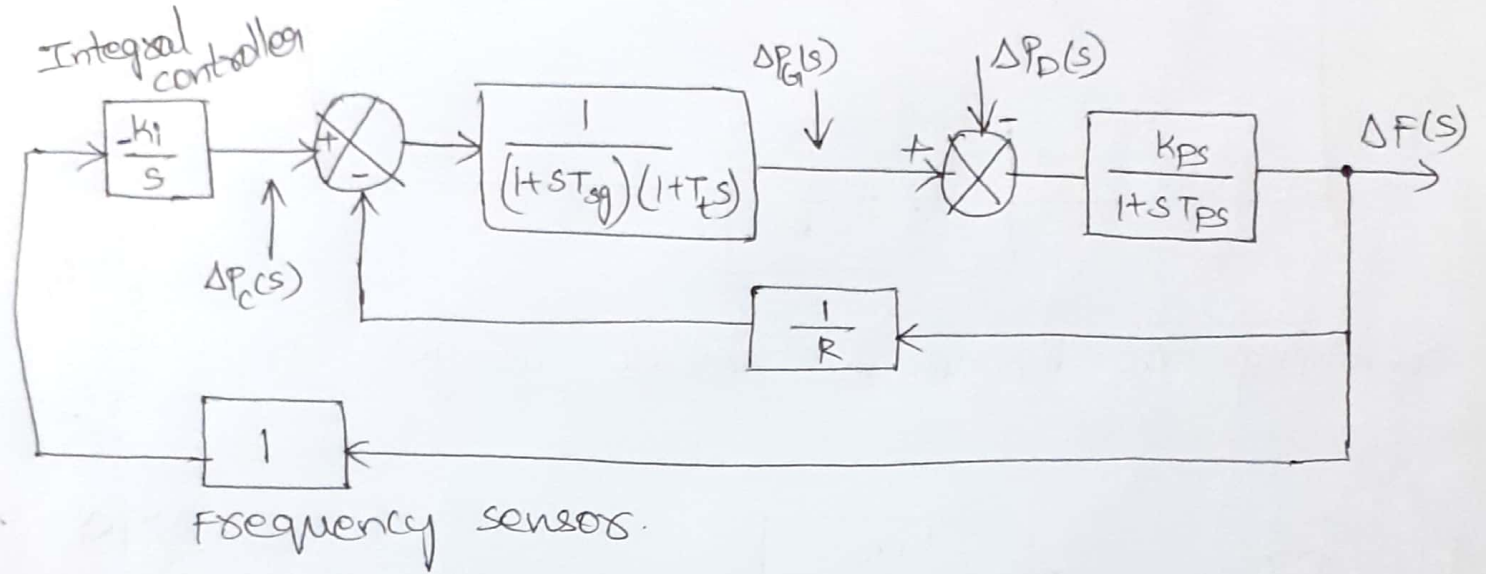
From dynamic response and steady state responses of single-area control system, with the speed governing system installed on each machine, the steady load frequency characteristic for a given speed changes setting has considerable droop, e.g. for the system being assumed. in dynamic response, the steady state droop in frequency will be 2.9 Hz. from no load to full load. (1 pu load).

system frequency specifications are rather stringent <sup>(uncompromising)</sup> and therefore, so much change in frequency can not be tolerated. In fact, it is ~~expected~~ expected that the steady change in frequency will be zero.

while steady state frequency can be brought back to the scheduled value by adjusting speed changes setting, the system could undergo intolerable dynamic frequency changes with changes in load. It leads to the natural suggestion that the speed changes setting be adjusted automatically by monitoring the frequency changes.



For this purpose, a signal from  $\Delta f$  is fed through an integrator to the speed changer resulting in the block diagram configuration shown in fig(i).



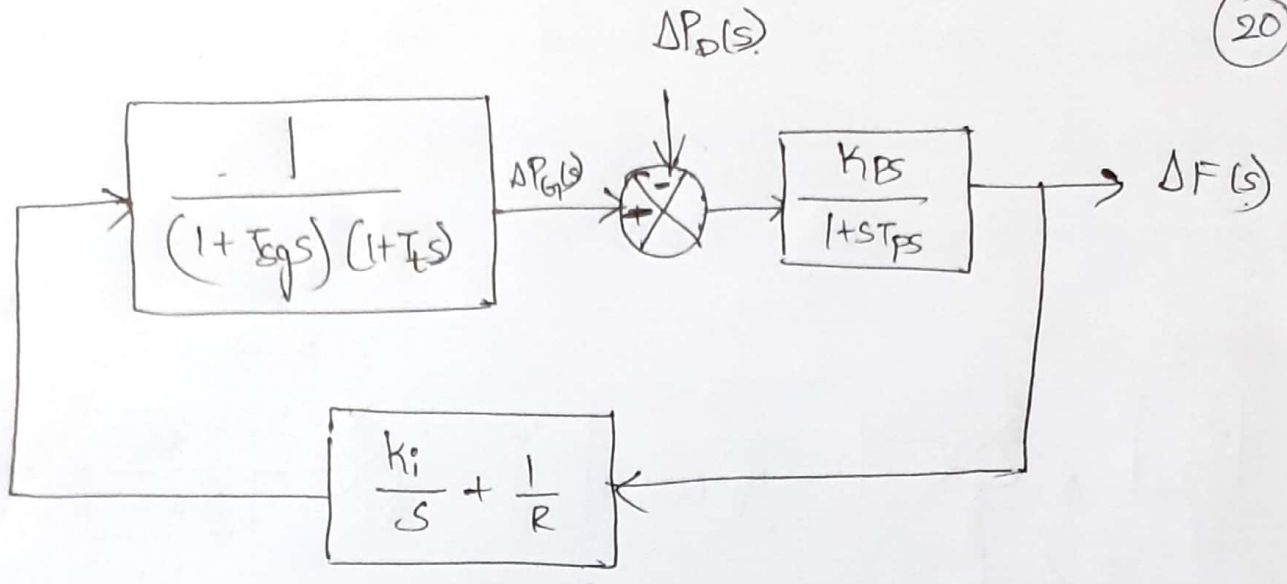
Fig(i) :- proportional plus integral load frequency control.

The system now modifies to a proportional plus integral controller, which, as is well known from control theory, gives zero steady state error. i.e.  $\Delta f|_{\text{steady state}} = 0$ .

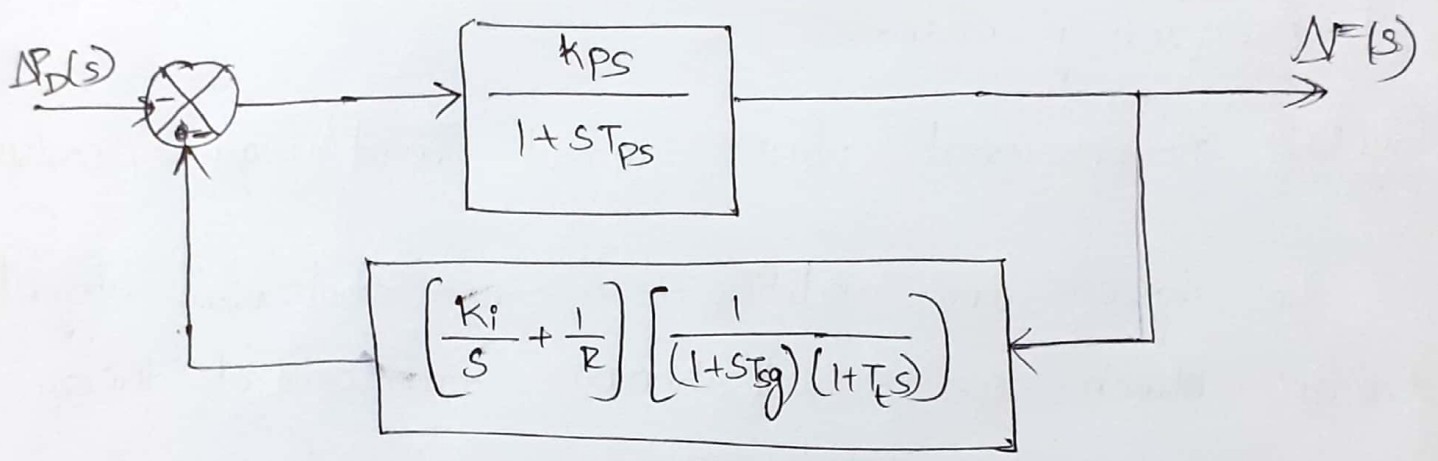
The signal  $\Delta f_c(s)$  generated by the integral control must be of opposite sign to  $\Delta f(s)$  which accounts for negative sign in the block for integral controller.

Now, 
$$\Delta f(s) = - \frac{k_P s}{(1+sT_P s) +}$$

By combining the blocks in parallel fig(i) becomes as follows



combining the blocks in series we get.



eliminating negative feedback loop

$$\Rightarrow \frac{\Delta F(s)}{-\Delta P_D(s)} = \frac{+ \left( \frac{K_{PS}}{1 + S T_{PS}} \right)}{1 + \left( \frac{K_i}{s} + \frac{1}{R} \right) \left( \frac{1}{(1 + S T_{sg}) (1 + T_t)} \right) \left( \frac{K_{PS}}{1 + S T_{PS}} \right)}$$

$$\Rightarrow \Delta F(s) = \frac{- K_{PS}}{(1 + S T_{PS}) + \left( \frac{K_i}{s} + \frac{1}{R} \right) \frac{K_{PS}}{(1 + S T_{sg}) (1 + S T_t)}} \times \frac{\Delta P_D}{s}$$



$$\Rightarrow \Delta F(s) = \frac{-k_{ps}}{\left(1 + sT_{ps}\right) + \left(\frac{Rk_i + s}{sR}\right) \left[\frac{k_{ps}}{(1 + sT_{sg})(1 + sT_t)}\right]} \times \frac{\Delta P_D}{s}$$

$$\Delta F(s) = \frac{-k_{ps}}{\left[\frac{s(1 + sT_{ps})(1 + sT_{sg})R + (Rk_i + s)k_{ps}}{s(1 + sT_{sg})(1 + sT_t)R}\right]} \times \frac{\Delta P_D}{s}$$

$$\Delta F(s) = \frac{Rk_{ps}s(1 + sT_{sg})(1 + sT_t)}{s(1 + sT_{sg})(1 + sT_{ps})R + (Rk_i + s)k_{ps}} \times \frac{\Delta P_D}{s}$$

→ eq(1)

Obviously, ~~with~~

$$\Delta f / \text{steady state} = \lim_{s \rightarrow 0} s \Delta F(s) = 0$$

→ eq(2)

∴ from eq(2) we find that the steady state change in frequency has been reduced to zero by the addition of integral controllers.

if 'df' reaches steady state (a constant value) only when  $\Delta P_c = \Delta P_D = \text{constant}$ . Because of the integrating action of the controller, this is only possible if  $\Delta f = 0$

In central load frequency control of a given control area, the change (error) is known as "Area Control Error" (ACE). The additional signal fed back in the modified control scheme presented above is the integral of ACE.

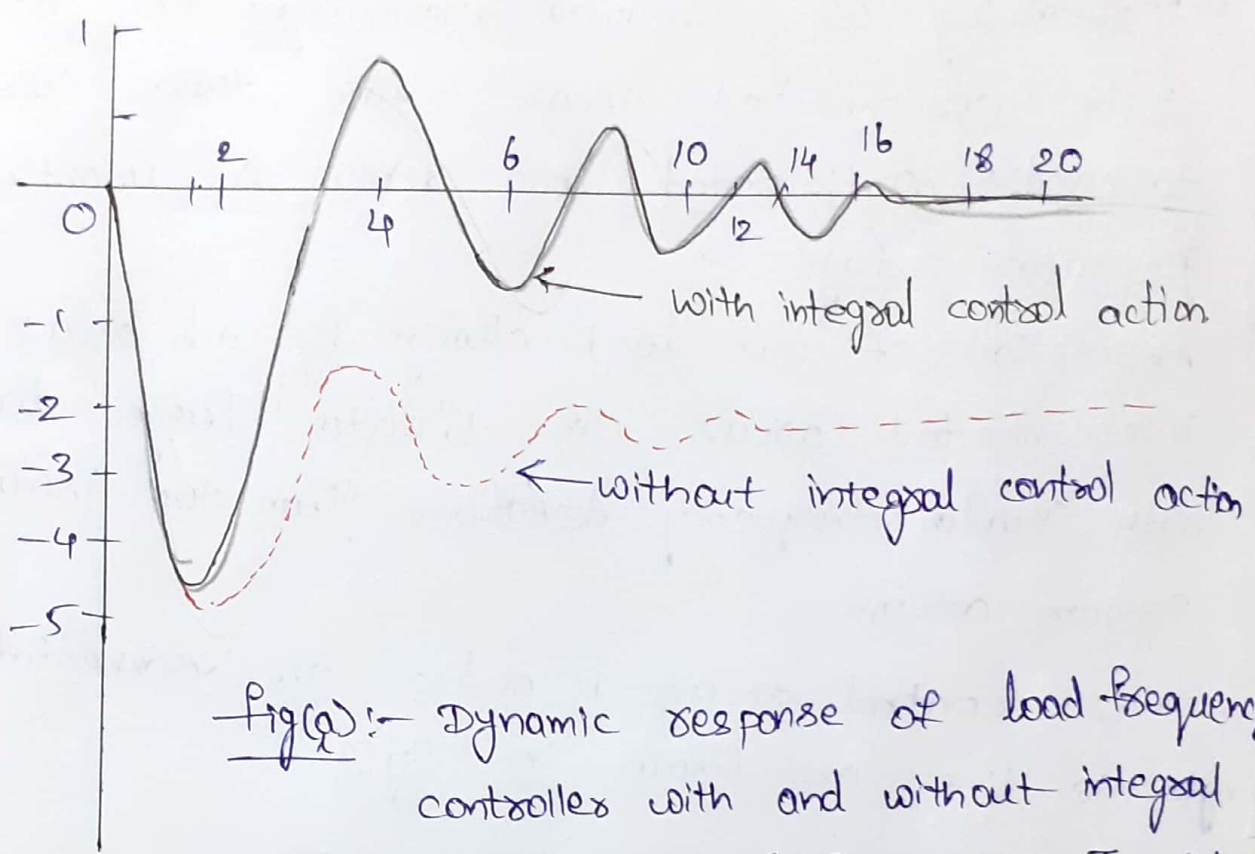
In the above scheme ACE being zero under steady conditions, a logical design criterion is the minimization of  $\int ACE dt$  for a step disturbance. This integral is indeed the "time error" of a synchronous electric clock run from the power supply.

In fact, modern power systems keep track of integrated time error all the time. A corrective action (manual adjustment  $\Delta P_c$ , the speed changer setting) is taken by a large (preassigned) station in the area as soon as the time error exceeds a prescribed value.

The dynamics of the proportional plus integral controllers can be studied numerically only, the system being of "fourth order" — the order of the system has increased by one with the addition of the integral loop. The dynamic response of the proportional plus integral controller with  $k_i = 0.09$  for a step load disturbance of 0.01 pu obtained through digital computers are plotted in fig(ii). For the sake of comparison the dynamic



response without integral controller action is also plotted on the same figure.



Fig(a) :- Dynamic response of load frequency controller with and without integral control action ( $\Delta P_D = 0.01 \text{ pu}$ ,  $T_{sg} = 0.4 \text{ sec}$ ,  $T_t = 0.5 \text{ sec}$ ,  $T_{ps} = 20 \text{ sec}$ ,  $K_{ps} = 100$ ,  $R = 3$ ,  $K_i = 0.09$ )

Load Frequency control of 2-Area system :-

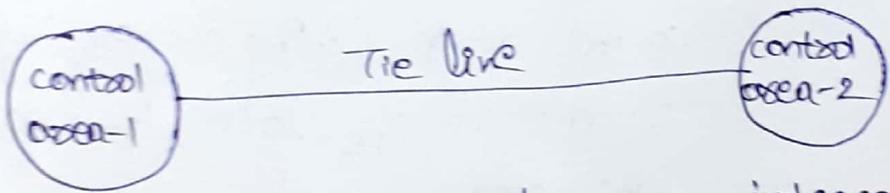
An extended power system can be divided into a number of load frequency control (LFC) areas, which are interconnected by tie lines. Such an operation is called "pool operation".

A power pool is an interconnection of the power systems of individual utilities. Each power system operates independently within its own jurisdiction, but there are contractual agreements dealing with the operating procedures to maintain system frequency.

The basic principle of a pool operation in the normal steady state provides:

- i) Maintaining of scheduled interchanges of tie-line power: the interconnected areas share their reserve power to handle anticipated load peaks & unanticipated generator outages.
- ii) Absorption of own load change by each area: the interconnected areas can tolerate larger load changes with smaller frequency deviations than the isolated power system areas.

Two control areas - 1 and 2 are connected by a single tie line as shown in Fig(a).



Fig(a): Two control areas interconnected through a single tie line.

Here, the control objective is to regulate the frequency of each area and to simultaneously regulate the power flow through the tie line according to an inter area power agreement.

In the case of an isolated control area, the zero steady-state error in frequency (i.e.  $\Delta f_{steady\ state} = 0$ ) can be obtained by using a proportional plus integral controller, whereas in two-control area case, proportional plus integral controller will be installed to give steady-state error in frequency.



(i.e.,  $\Delta P_{\text{steady state}} = 0$ ) in tie-line power flow (i.e.,  $\Delta P_{TL} = 0$ ) (25)

in addition to zero steady-state error in frequency.

Each control area is represented by an equivalent turbine, generator, and governor system.

In case of a single control area, the incremental power ( $\Delta P_G - \Delta P_D$ ) was considered by the rate of increase of stored kinetic energy and increase in area load caused by the increase in frequency.

In a two-area case, the tie-line power must be accounted for the incremental power balance equation of each area, since there is power flow in or out of the area through the tie line.

Power flow out of control area-1 can be expressed as

$$P_{\text{tie},1} = \frac{|V_1| |V_2|}{X_{12}} \sin(\delta_1 - \delta_2) \quad \text{--- eq (1)}$$

where  $\delta_1, \delta_2$  = power angles of equivalent machines of the two areas.

For incremental changes in ' $\delta_1$ ' & ' $\delta_2$ ' the incremental tie line power can be expressed as

$$\Delta P_{\text{tie},1} (\text{pu}) = T_{12} (\delta_1 - \Delta \delta_2)$$

$|V_1| |V_2|$  are voltage magnitudes of Area-1 & Area-2

$X_{12}$  is the tie-line reactance.

If there is change in load demands of two areas,

these will be incremental changes in power angles ( $\Delta\delta_1$  &  $\Delta\delta_2$ ). then, the change in the tie-line power is

$$\begin{aligned}
 P_{tie,1} + \Delta P_{tie,1} &= \frac{|V_1||V_2|}{X_{12}} \sin\left[(\delta_1 - \delta_2) + (\Delta\delta_1 - \Delta\delta_2)\right] \\
 &= \frac{|V_1||V_2|}{X_{12}} \left[ \sin(\delta_1 - \delta_2) \cdot \cos(\Delta\delta_1 - \Delta\delta_2) + \right. \\
 &\quad \left. \cos(\delta_1 - \delta_2) \cdot \sin(\Delta\delta_1 - \Delta\delta_2) \right] \\
 &= \frac{|V_1||V_2|}{X_{12}} \left[ \sin(\delta_1 - \delta_2) + \cos(\delta_1 - \delta_2) (\Delta\delta_1 - \Delta\delta_2) \right] \quad \left\{ \because \sin(A+B) = \sin A \cos B + \sin B \cos A \right\} \\
 &= \frac{|V_1||V_2|}{X_{12}} \sin(\delta_1 - \delta_2) + \frac{|V_1||V_2|}{X_{12}} \left[ \cos(\delta_1 - \delta_2) (\Delta\delta_1 - \Delta\delta_2) \right] \quad \left\{ \because \Delta\delta_1 - \Delta\delta_2 \approx 0 \right\}
 \end{aligned}$$

therefore, change in incremental tie-line power can be expressed as

$$\Delta P_{tie,1} = \frac{|V_1||V_2|}{X_{12}} \cos(\delta_1 - \delta_2) (\Delta\delta_1 - \Delta\delta_2)$$

$$\Rightarrow \boxed{\Delta P_{tie,1} (P.U) = T_{12} (\Delta\delta_1 - \delta_2)} \quad \rightarrow \text{eq (2)}$$

where  $T_{12} = \frac{|V_1||V_2|}{X_{12} P_1} \cos(\delta_1 - \delta_2) =$  synchronising coefficient.

(or)  
stiffness coefficient.

$$\Rightarrow \boxed{T_{12} = \frac{P_{max,12}}{P_1} \cos(\delta_1 - \delta_2)} \quad \rightarrow \text{eq (3)}$$



where,  $P_{max,12} = \frac{|V_1| |V_2|}{X_{12}}$  = static transmission capacity of the tie line. (27)

considers the change in frequency as

$$\Delta \omega = \frac{d}{dt} (\Delta \delta)$$

$$\Rightarrow 2\pi \Delta f = \frac{d}{dt} (\Delta \delta)$$

$$\Delta f = \frac{1}{2\pi} \times \frac{d}{dt} (\Delta \delta) \text{ Hz}$$

In other words,  $\frac{d}{dt} (\Delta \delta) = 2\pi \Delta f$

$$\int \frac{d}{dt} (\Delta \delta) = \int 2\pi \Delta f$$

$$\Rightarrow \Delta \delta = 2\pi \int \Delta f dt \text{ radians.}$$

Hence, the changes in power angles for areas-1 & 2 are

$$\left. \begin{array}{l} \Delta \delta_1 = 2\pi \int \Delta f_1 dt \\ \Delta \delta_2 = 2\pi \int \Delta f_2 dt \end{array} \right\} \longrightarrow \text{eq (3)}$$

Since the incremental power angles are related in terms of integrals of incremental frequencies eq(2) can be modified as

$$\Delta P_{tie,1} = T_{12} \left[ 2\pi \int \Delta f_1 dt - 2\pi \int \Delta f_2 dt \right]$$

$$\Delta P_{tie,1} = 2\pi T_{12} \left[ \int \Delta f_1 dt - \int \Delta f_2 dt \right] \longrightarrow \text{eq (4)}$$

$\Delta f_1$  &  $\Delta f_2$  are the incremental frequency changes of areas-1 and 2 respectively.

By, the incremental tie-line power out of Area-2 is (28)

$$\Delta P_{Tie,2} = 2\pi T_{21} \left[ \int \Delta f_2 dt - \int \Delta f_1 dt \right]$$

where → eq (5)

$$T_{21} = \frac{|V_1| |V_2|}{x_{21} P_2} \cos(\delta_2 - \delta_1)$$

→ eq (6)

eq (6) ÷ eq (3), we get

$$\frac{T_{21}}{T_{12}} = \frac{\left( \frac{|V_1| |V_2|}{x_{21} P_2} \right)}{\left( \frac{|V_1| |V_2|}{x_{12} P_1} \right)} = \frac{P_1}{P_2} = a_{12} \quad \left\{ \because x_{21} = x_{12} \right\}$$

$$\therefore \frac{T_{21}}{T_{12}} = a_{12} \Rightarrow T_{21} = a_{12} T_{12}$$

and hence,  $\Delta P_{Tie,2} = a_{12} \Delta P_{Tie,1}$  → eq (7)

From LFC-1, surplus power can be expressed in p.u. is

$$\Delta P_G - \Delta P_D = \frac{2H}{f_0} \frac{d}{dt} (\Delta f) + B \Delta f \quad (\text{for a single area case})$$

For a 2-Area case, the surplus power can be expressed in p.u. as

$$\Delta P_{G1} - \Delta P_{D1} = \frac{2H_1}{f_0} \frac{d}{dt} (\Delta f_1) + B_1 \Delta f_1 + \Delta P_{Tie,1}$$

→ eq (8)



Taking Laplace transform on both sides of eq (8) (29)  
 we get

$$\Delta P_{G1}(s) - \Delta P_{D1}(s) = \frac{2H_1}{f_0} s (\Delta F_1(s)) + B_1 \Delta F_1(s) + \Delta P_{Tie,1}(s)$$

rearranging the above eqn as follows, we get

$$\Delta P_{G1}(s) - \Delta P_{D1}(s) = \Delta F_1(s) \left[ \frac{2H_1}{f_0} s + B_1 \right] + \Delta P_{Tie,1}(s)$$

$$\Rightarrow \Delta F_1(s) \left[ \frac{2H_1}{f_0} s + B_1 \right] = \left[ \Delta P_{G1}(s) - \Delta P_{D1}(s) \right] - \Delta P_{Tie,1}(s)$$

$$\Delta F_1(s) = \frac{\left[ \Delta P_{G1}(s) - \Delta P_{D1}(s) - \Delta P_{Tie,1}(s) \right]}{\left[ \frac{2H_1}{f_0} s + B_1 \right]}$$

$$= \left[ \Delta P_{G1}(s) - \Delta P_{D1}(s) - \Delta P_{Tie,1}(s) \right] \times \frac{1}{B_1 \left[ 1 + \left( \frac{2H_1}{B_1 f_0} \right) s \right]}$$

$$= \left[ \Delta P_{G1}(s) - \Delta P_{D1}(s) - \Delta P_{Tie,1}(s) \right] \times \left[ \frac{\left( \frac{1}{B_1} \right)}{1 + \left( \frac{2H_1}{B_1 f_0} \right) s} \right]$$

$$\Delta F_1(s) = \left[ \Delta P_{G1}(s) - \Delta P_{D1}(s) - \Delta P_{Tie,1}(s) \right] \left[ \frac{k_{ps1}}{1 + s T_{ps1}} \right]$$

where,  $k_{ps1} = \frac{1}{B_1}$

$$T_{ps1} = \frac{2H_1}{B_1 f_0}$$

→ eq (9)

From Single Area control, we have LFC  
 comparing, the generator load model equation for ~~the~~

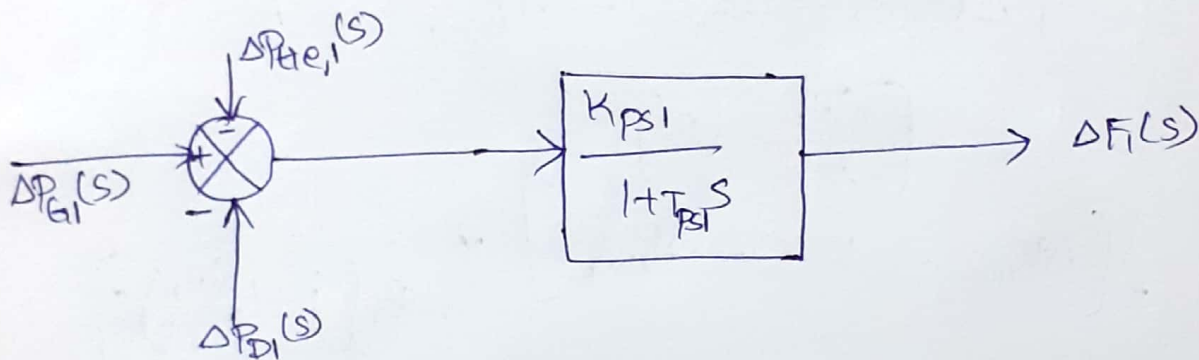
$$\Delta F_1(s) = \left[ \Delta P_{G1}(s) - \Delta P_{D1}(s) \right] \times \left[ \frac{k_{ps1}}{1 + T_{ps1}s} \right], \text{ only change}$$

is the appearance of the signal  $\Delta P_{tie,1}(s)$  as shown in fig(b), the signal  $\Delta P_{tie,1}(s)$  is obtained by taking Laplace transform of eq(4) we get

$$L[\Delta P_{tie,1}(s)] = 2\pi T_{12} \text{LT}\left[\int \Delta f_1 dt - \int \Delta f_2 dt\right]$$

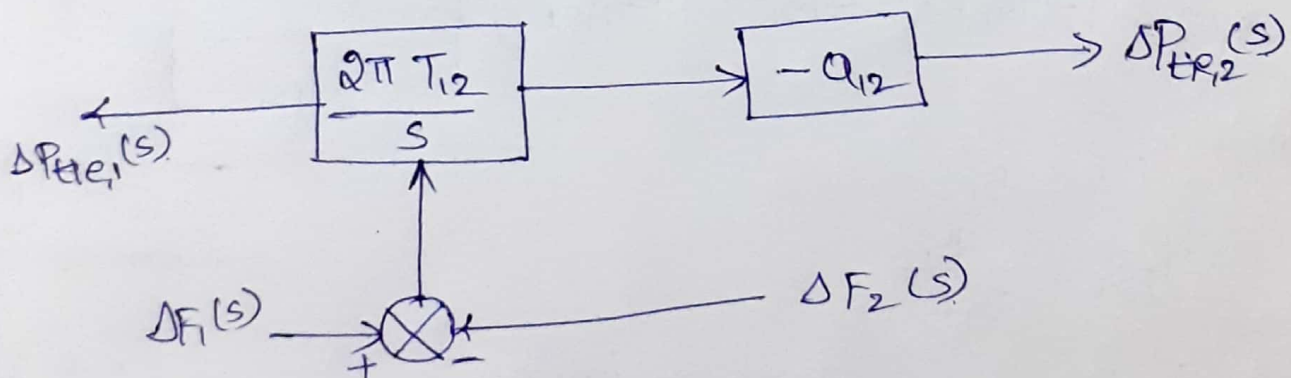
$$\Delta P_{tie,1}(s) = 2\pi T_{12} \left[ \frac{\Delta F_1(s)}{s} - \frac{\Delta F_2(s)}{s} \right]$$

$$\Delta P_{tie,1}(s) = \frac{2\pi T_{12}}{s} [\Delta F_1(s) - \Delta F_2(s)] \rightarrow \text{eq(10)}$$



Fig(b):-

The corresponding block diagram is shown in Fig(c).



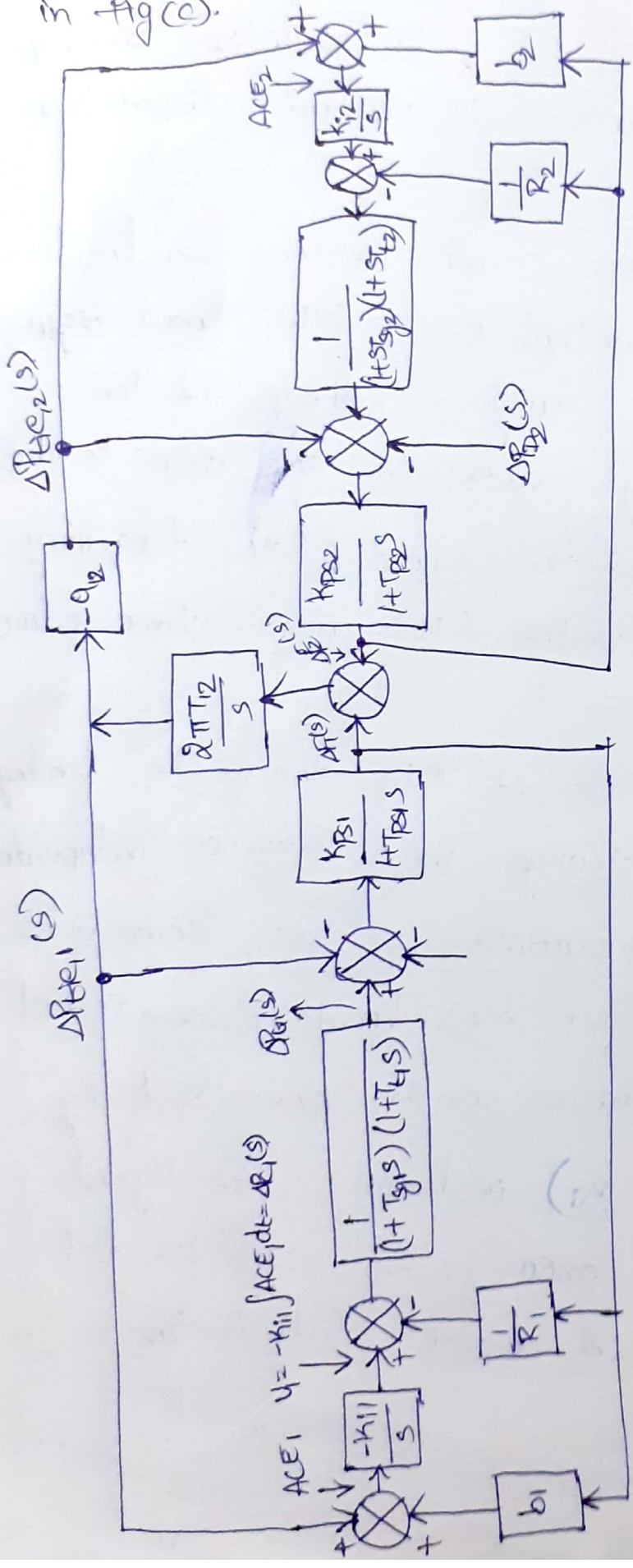
Fig(c):-

For the control area-2,  $\Delta P_{tie,2}(s)$  is given by eq(11)



$$\Delta P_{tie,12}(s) = \frac{-2\pi a_0 T_{12}}{s} \left( \Delta F_1(s) - \Delta F_2(s) \right)$$

The block diagram representation of eq(10) & eq(11) is shown in fig(c).



fig(c):- composite block diagram of two area load frequency control (lfb loops provided with integral of respective area control errors).

## Economic Dispatch control :-

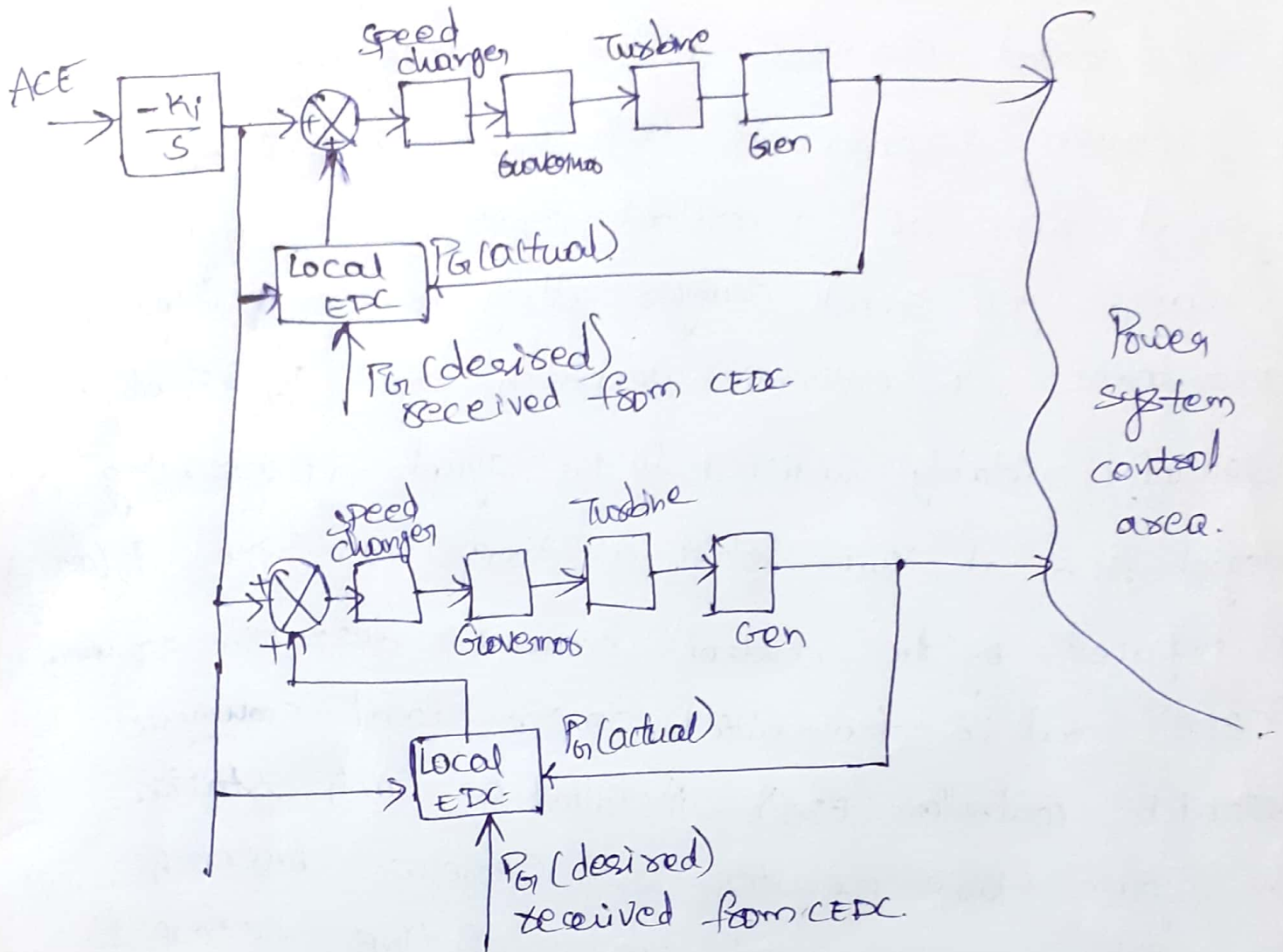
(32)

Load frequency control with integral controller achieves zero steady state frequency error & a fast dynamic response, but it exercises no control over the relative loading of various generating stations (i.e. economic dispatch) of the control area.

For example, if a sudden small increase in load (say 1%) occurs in the control area, the load frequency control changes the speed change settings of the governors of all generating units of the area so that together these units match the load & the frequency returns to the scheduled value (this action takes place in few seconds).

However, in the process of this change the loading of various generating units change in a manner independent of economic loading considerations. In fact, some units in the process may even get over loaded. Some control over loading of individual units can be exercised by adjusting the gain factors ( $K_i$ ) included in the signal representing integral of the area control error as fed to individual units. However, this is not satisfactory.





EDC — Economic dispatch controller  
 CEDC — central Economic dispatch controller

Fig (a) :- control area load frequency & economic dispatch control.

A satisfactory solution is achieved by using independent controls for load frequency & economic dispatch. While the load frequency controller is a fast acting control (a few seconds) and regulates the system around an operating point, the economic dispatch controller is a slow acting control, which adjusts the speed changer setting every minute (or half a

minute) in accordance with a command signal generated by the central economic despatch computer. Fig (C) gives the schematic diagram of both these controls for two typical units of a control area. The signal to change the speed changes setting is constructed in accordance with economic despatch error  $(P_1(\text{desired}) - P_1(\text{actual}))$ , suitably modified by the signal representing integral ACE at that instant of time. The signal  $P_2(\text{desired})$  is computed by the central economic despatch computer (CEDC) and is transmitted to the local economic despatch controller (EDC) installed at each station. The system thus operates with economic despatch error only for very short periods of time before it is readjusted.

### Tie-line Bias control

The speed-changes command signals will be obtained from the block-diagram shown in fig.

$$\Delta P_{c1} = -k_{i1} \int (\Delta P_{tie,1} + b_1 \Delta f_1) dt \rightarrow (1)$$

$$\Delta P_{c2} = -k_{i2} \int (\Delta P_{tie,2} + b_2 \Delta f_2) dt \rightarrow (2)$$

The constants  $k_{i1}$  &  $k_{i2}$  are the gains of the integrators. The first terms on the right-hand side of eq(1) & eq(2) constitute & are known as tie-line bias controls. It is observed that  $f_{os}$  decreases in both frequency & tie-line



Power, the speed-changer position decreases and hence (35) the power generation should decrease, i.e., if the ACE is negative, then the area should increase its generation

So, the right-hand side terms of eq (1) & eq (2) are assigned a negative sign

### AREA CONTROL ERROR - TWO-AREA CASE :-

In a single-area case, ACE is the change in frequency. The steady-state errors in frequency will become zero (i.e.  $\Delta f_{ss} = 0$ ) when ACE is used in the integral-control loop.

In a two area case, ACE is the linear combination of the change in frequency and change in tie-line power. In this case to make the steady-state tie-line power zero (i.e.  $P_{tie} = 0$ ), another integral-control loop for each area must be introduced in addition to the integral frequency loop to integrate the incremental tie-line power signal and feed it back to the speed-changer. Thus for control area-1, we have

$$\boxed{ACE_1 = \Delta P_{tie,1} + b_1 \Delta f_1} \quad \longrightarrow \text{eq (1)}$$

where,  $b_1 = \text{constant} = \text{area frequency bias}$ .

Taking L.T eq (1) we get

$$\boxed{ACE_1(s) = \Delta P_{tie,1}(s) + b_1 \Delta f_1(s)}$$

Similarly for control area-2, we have  $\boxed{ACE_2(s) = \Delta P_{tie,2}(s) + b_2 \Delta f_2(s)}$

## PROBLEMS :-

1) A single-axis system has the following data:

Speed-regulation,  $R = 4 \text{ Hz/P.U.MW}$

Damping coefficient,  $B = 0.1 \text{ P.U.MW/Hz}$

Power System

psoc by

Siva nagaraj

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REACTIVE POWER CONTROLOVER VIEW OF REACTIVE POWER CONTROL:-

Power industry is required to generate both real and reactive power. Reactive power is required to excite various equipment in addition to energizing the transmission network. The reactive power requirement of the consumers rises mostly from lagging Vars to supply magnetizing current to transformers and induction motors.

In the transmission network requirement is the difference b/w that absorbed in the series inductance ( $I^2X$ ) and that produced in the shunt capacitance ( $V^2B$ ). There is a level of loading at which the leading Vars of the charging current balance the lagging Vars of the inductive lines and is called the Natural (or) surge impedance loading of the system.

This natural load for a transmission line is given approximately as  $\left(\frac{B}{X}\right)$  P.V. In case of cables the shunt capacitance is higher and series inductance is lower. As a consequence of which the natural loading is higher. However, this limit is generally above the thermal limit of the cable. The conductors are bundled together two together for 220 kV & four for 380 kV.

The reactive power compensation of the transmission system depends on the load and its power factor. When the line is operated at no load, the full charging power occurs & would result in considerable increase in voltage unless some compensating device is used. With full compensation at no load, the line may be operated at any partial load between no load and full load with the voltage not exceeding the permissible limits. (2)

Higher voltages are selected for transmission to keep the losses in an economically justifiable relationship to the power  $P$ . In view of the inverse square relationship, reduction of reactive power becomes an essential factor for obtaining efficient operation of high voltage lines.

In order to supply quality service to customers reliably & economically voltage or var control plays a leading role. Such a control has to be exercised at all in the power system i.e. right from the generating point to the consumer terminals.

Rapid changes in voltage (flicker) can result due to some industrial loads such as arc furnaces, arc welders, and wood chippers. Even slow changes in power demand following the daily load cycle supplying a composite system



load may create relatively large variations in voltage if control is not exercised. Also, there may be cyclic and non-cyclic loads that create voltage disturbance at both transmission and distribution level. (3)

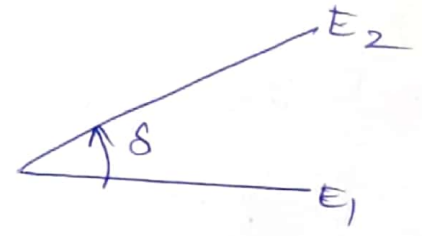
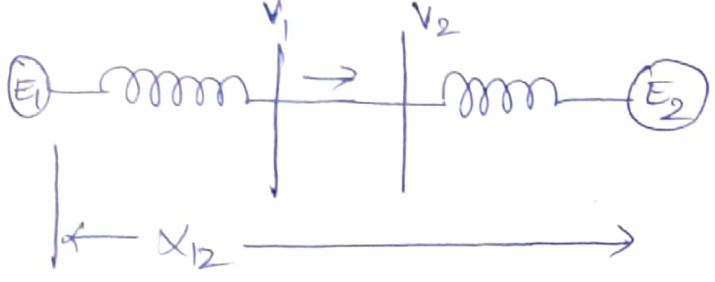
In addition, events such as planned line switching unplanned line trips planned and unforeseen generators trips and equipment failure may produce voltage & VAR variations. Unless proper voltage support is given at strategic locations in the system, the aforesaid events may result in loss of stability and possible loss of service to a large number of consumers.

For long distance transmission of power, the use of HVDC transmission has proved economical in certain cases. The VAR demand of DC terminals varies usually from 0-60% of the MW rating of the DC line as power transfer is varied over its full range. When a fault takes place on the near by AC system, the VAR demand of the DC line may reach a high value and unless compensated may produce large AC voltage variations.

Continuous control can be achieved by means of synchronous compensators installed at line ends and also in the intermediate substations.

The use of shunt connected controllable VAR compensation to improve the power transfer capability

and stability is an acknowledged fact. From fig(i) & fig(ii) it can be seen that the theoretical maximum power transfer takes place at a power angle of  $\delta_{12} = 90^\circ$



$$P_{12} = \frac{E_1 E_2}{X_{12}} \sin \delta_{12}$$

Fig (i):-

Fig (ii):-

With an intermediate, controllable, shunt Vars compensator, the angle ~~also~~ could be increased, in principle to 180 across the line fig (iii) & fig (iv):-

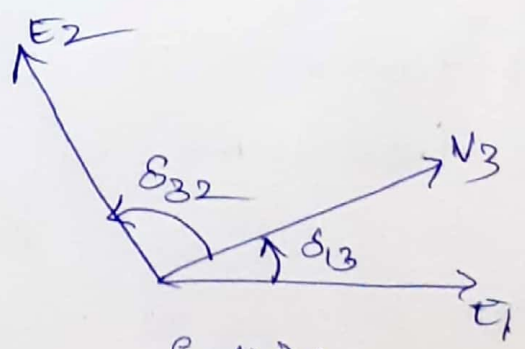
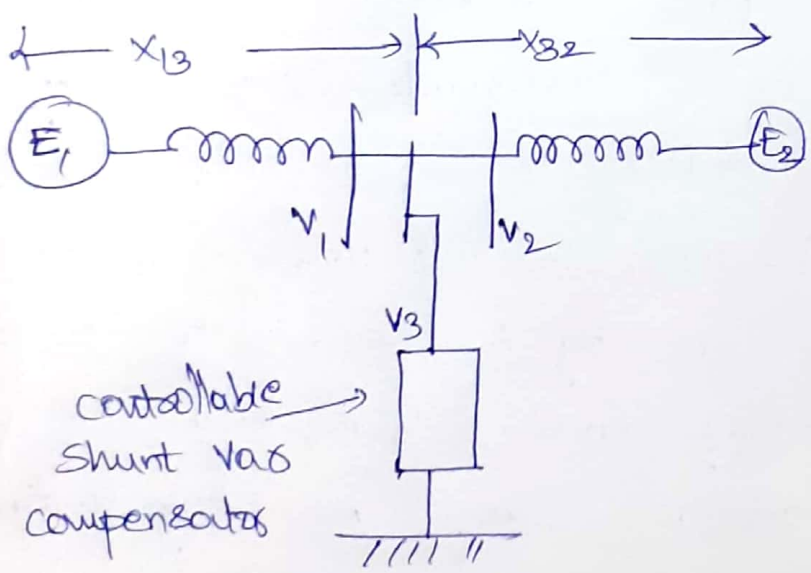


Fig (iv):-

Fig (iii):-

$$P_{13} = \frac{E_1 \cdot V_3}{X_{13}} \sin \delta_{13}$$
$$P_{32} = \frac{V_3 E_2}{X_{13}} \sin \delta_{32}$$



and stability ~~is~~

(5)

## REACTIVE POWER COMPENSATION IN TRANSMISSION SYSTEMS :-

(OR)

### Compensated transmission lines

The change in the electrical characteristics of a transmission line in order to increase its power transmission capability is known as "line compensation".

While satisfying the requirements for a transmission system (i.e. synchronism, voltages must be kept near their rated values etc.) a compensation system ideally performs the following functions:

- 1) It provides the flat voltage profile at all levels of power transmission.
- 2) It improves the stability by increasing the maximum transmission capacity.
- 3) It meets the reactive power requirements of the transmission system economically.

The following types of compensations are generally used for transmission lines:

- i) Virtual -  $Z_0$
- ii) Virtual -  $\theta$
- iii) compensation by sectioning.

i) Virtual- $z_0$  :- compensation of line, by which the uncompensated surge impedance  $z_0$  is modified to virtual surge impedance  $z'_0$ , is called "Virtual surge impedance compensation or Virtual  $z_0$  compensation"

ii) Virtual- $\theta$  :-

Once a line is computed for  $z_0$ , the only way to improve stability is to reduce the effective value of  $\theta$ . Two alternative compensation strategies have been developed to achieve this

a) Apply series capacitors to reduce  $X_L$  & thereby reduce  $\theta$ , since  $\theta = \beta l = \omega \sqrt{LC} l = \sqrt{\frac{X_L}{X_C}}$  at fundamental frequency. This method is called line-length compensation (or)  $\theta$ -compensation.

iii) compensation by sectioning :-

Divide the line into shorter sections that are more (or) less independent of one another. This method is called compensation by sectioning. It is achieved by connecting constant voltage compensations at intervals along the line.

ADVANTAGES :-  
the effectiveness of a compensated system



is Gauged by the product of the line length and maximum transmission power capacity. (7)

2) compensated lines enable the transmission of the natural load over longer distances, & shorter compensated lines can carry loads more than natural load.

3) the flat voltage profile can be achieved if the effective surge impedance of the line is modified as to a virtual value  $Z_0'$ , for which the corresponding virtual natural load  $\left[ \frac{V^2 (KV)}{Z_0'} \right] = \text{actual load}$ .

4) the surge impedance of the uncompensated line is

$$Z_0 = \sqrt{\frac{L}{C}}, \text{ which can be written as } \sqrt{x_L x_C},$$

if the series and /or the shunt reactance  $x_L$  &  $x_C$  are modified, respectively. Then, the line can be made to have virtual surge impedance  $Z_0'$  & a virtual natural load  $P'$  for which

$$P' = \frac{V^2 (KV)}{Z_0'}$$

COMPARISON OF DIFFERENT TYPES OF COMPENSATING EQUIPMENT FOR TRANSMISSION SYSTEMS :-

(08)

# TYPES OF COMPENSATING EQUIPMENT FOR TRANSMISSION

## SYSTEMS :-

Compensating equipment	Advantages	Disadvantages
1) switched shunt reactors	- simple in principle and construction	- Fixed in value.
2) switched shunt-capacitors	- simple in principle and construction	- Fixed in value - switching transients. Required over voltage protection and sub-harmonic filters. Limited overload capacity.
3) series capacitors	- Simple in principle. Performance relatively sensitive to location. Has useful overload capability	- High-maintenance requirements slow control response.
4) synchronous condensers	- Fully controllable. Low harmonics	- performance sensitive to location requires strong foundations.
5) polyphase saturated reactors	- Very rugged construction. Large overload capability. No effect on fault level. Low harmonics	- Essentially fixed in value. performance sensitive to location and noisy.



compensating equipment

Advantages

Disadvantages

6) TCR

- Fast response. Fully controllable. No effect on fault level. can be rapidly repaired after failures

- Generator harmonics performance sensitive to location.

7) TSC

- can be rapidly repaired after failures. No harmonics

- No inherent ability capability to limit over voltages. complex bus work and controls low-frequency response with system. performance sensitive to location.

LOAD COMPENSATION :-



Here, the supply system is modeled as a thevenin's equivalent circuit with reactive power requirements. The compensator is modeled as a variable impedance/as a variable source (or sink) of reactive current/power. According to requirements, the selection of model used for each component can be varied.

- the assumption made in developing the relationships b/w supply system, the load, and the compensator is that the

the load and system characteristics are static/constant (or) changing slowly so that phasor representation can be used. (10)

1) power factor correction :-

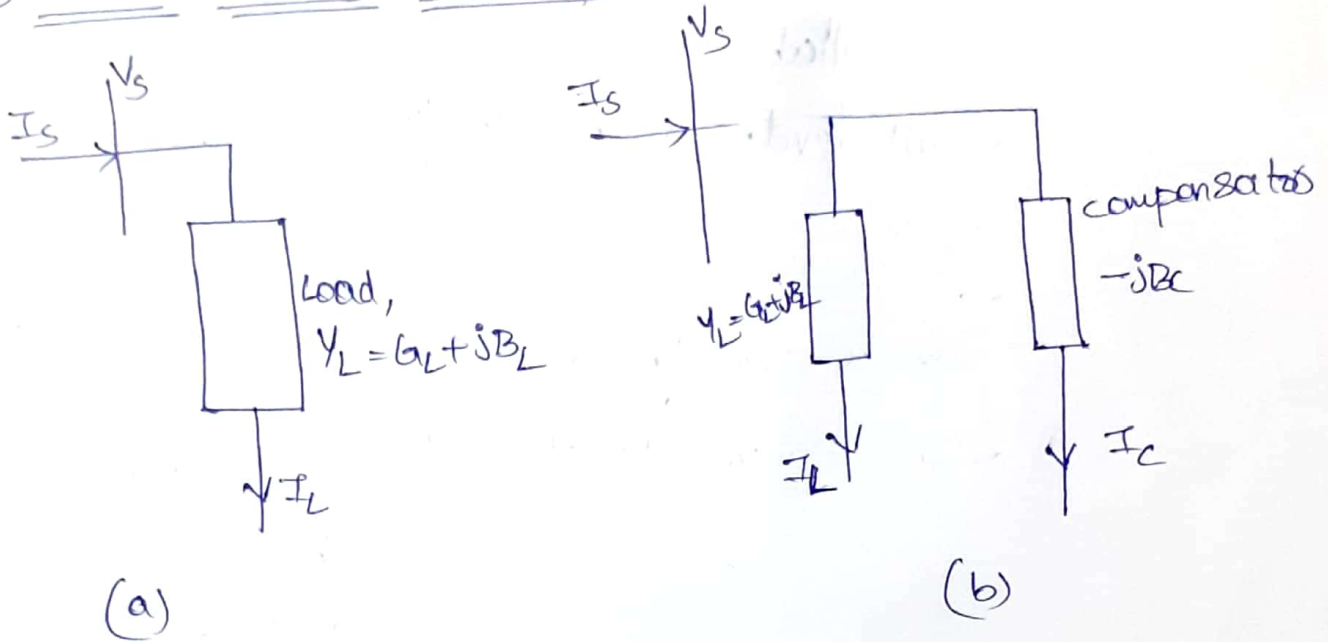


fig :- Representation of single-phase load ;

- a) without compensation
- b) with compensation.

considers a single-phase load with admittance

$Y_L = G_L + jB_L$  with a source voltage as shown in

fig(a). the load current ' $I_L$ ' is given by

$$I_L = V_s (G_L + jB_L) = V_s G_L + jV_s B_L = I_a + jI_b$$

where,

$I_a$  is the active component of the load current

$I_b$ , the reactive " " " " "

Apparent power of the load,  $S_L = V_s I_L^*$

$$= V_s^2 G_L - jV_s^2 B_L$$

$$S_L = P_L + jQ_L$$



where,  $P_L =$  the active power of the load  
 $Q_L =$  the reactive power of the load.

For inductive loads,  $B_L$  is -ve  
 $Q_L$  is +ve by convention.

the current supplied to the load is larger than when it is necessary to supply the active power alone by the factor

$$\frac{I_L}{I_a} = \frac{1}{\cos\phi_L} \quad \left\{ \therefore I_a = I_L \cos\phi_L \right\}$$

the objective of the p.f. correction is to compensate for the reactive power i.e. locally providing a compensator having a purely reactive admittance ' $jB_c$ ' in parallel with the load as shown in fig(b).

the current supplied from the source with the compensator is

$$I_s = I_L + I_c$$

$$= V_s (G_L + jB_L) - V_s (jB_c)$$

$$\boxed{I_s = V_s G_L = I_a} \quad \left\{ \therefore B_L = B_c \right\}$$

which makes the p.f. to unity, since ' $I_a$ ' is in phase with the source voltage  $V_s$ .

the current of the compensator,  $I_c = V_s Y_c = -jV_s B_c$

the apparent power of the compensator,  $S_c = V_s I_c^*$

$$\boxed{S_c = jV_s^2 B_c = -jQ_c}$$

$\left\{ \therefore S_c = P_c - jQ_c, \text{ for pure compensation, } P_c = 0 \right\}$

W.K.T,  $\boxed{Q_L = P_L \tan\phi_L}$

For a fully compensated system i.e.  $Q_L = Q_C$

(12)

$$\therefore Q_C = S_L \sin \phi_L$$

$$= S_L \sqrt{1 - \cos^2 \phi_L} = \left. \begin{array}{l} \therefore \sin^2 \phi_L + \cos^2 \phi_L = 1 \end{array} \right\}$$

The degree of compensation is decided by an economic trade-off b/w the capital cost of the compensator and the savings obtained by the reactive power compensation of the supply system over a period of time.

## 2) Voltage Regulation:-

It is defined as the proportional change in supply voltage magnitude associated with a defined change in load current, i.e., from no-load to full load. It is caused by the voltage drop in the supply impedance carrying the load current.

When the supply system is balanced, it can be represented as single-phase model as shown in fig(ii)(a)

The regulation is given by  $\frac{|V_S| - |V_L|}{|V_L|}$ ,

where,  $|V_L|$  is the load voltage.

### a) without compensator:-

From the phasor diagram of an uncompensated system, shown in fig(ii, b). the change in voltages is given by

$$\Delta V = V_S - V_L = Z_S I_L \longrightarrow \textcircled{1}$$

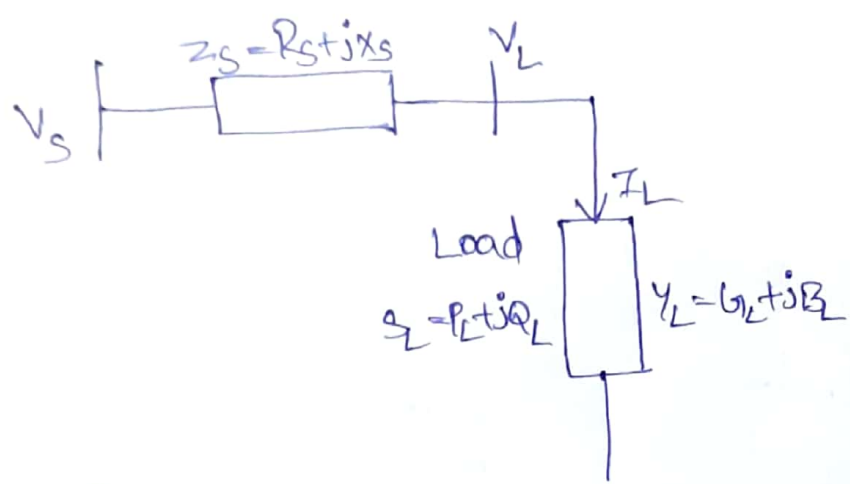


where  $Z_s = R_s + jX_s$  and

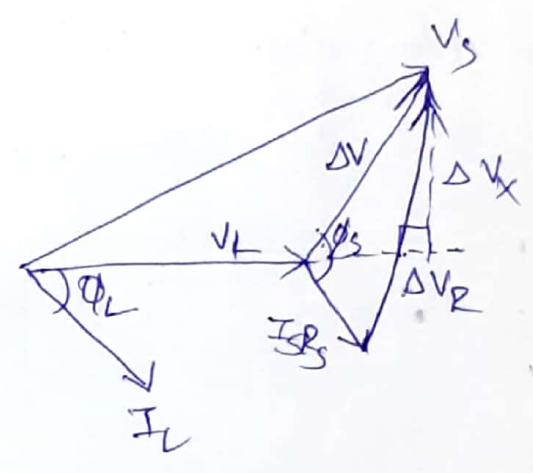
the load current,  $I_L = \frac{P_L - jQ_L}{V_L} \rightarrow \text{eq(2)}$

Substituting  $Z_s$  &  $I_L$  in eq(1), we get

$$\therefore \Delta V = (R_s + jX_s) \left[ \frac{P_L - jQ_L}{V_L} \right]$$



fig(a):-



fig(b):-

- fig(ii):-
- (a) circuit model of an uncompensated load and supply system
  - (b) phasor diagram for an uncompensated system.

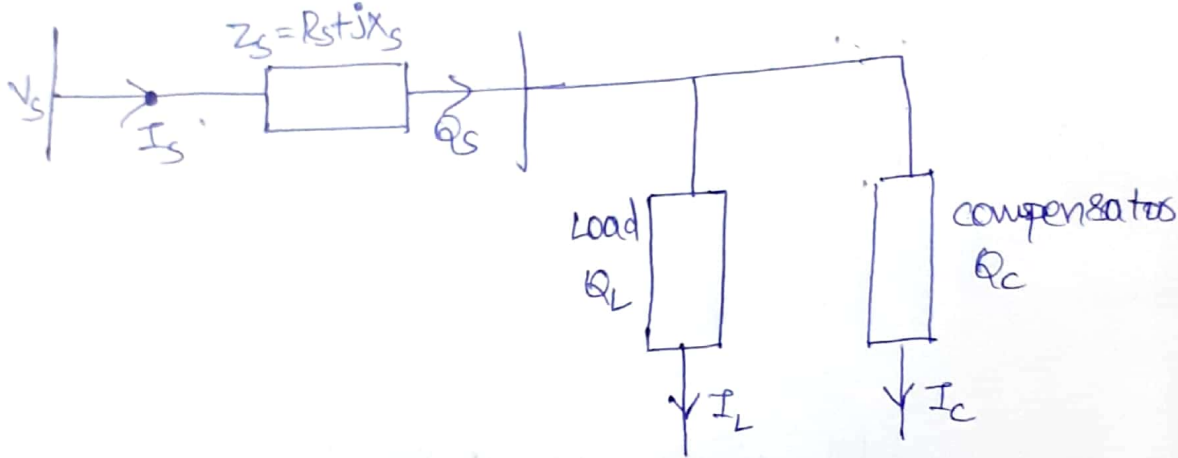
$$\Rightarrow \Delta V = \frac{R_s P_L - jQ_L R_s + jX_s P_L - (-j)X_s Q_L}{V_L}$$

$$= \frac{R_s P_L + X_s Q_L}{V_L} - j \frac{(X_s P_L - Q_L R_s)}{V_L}$$

considering the line parameters to be constant.

b) With compensator :-

In this case, a purely reactive compensator is connected across the load as shown in fig (iii, a). To make the voltage regulation zero i.e., the supply voltage ( $|V_s|$ ) equals the load voltage ( $|V_L|$ ). The corresponding phasor diagram is shown in fig (iii, b).



Fig(a): circuit model of a compensated load & supply system.

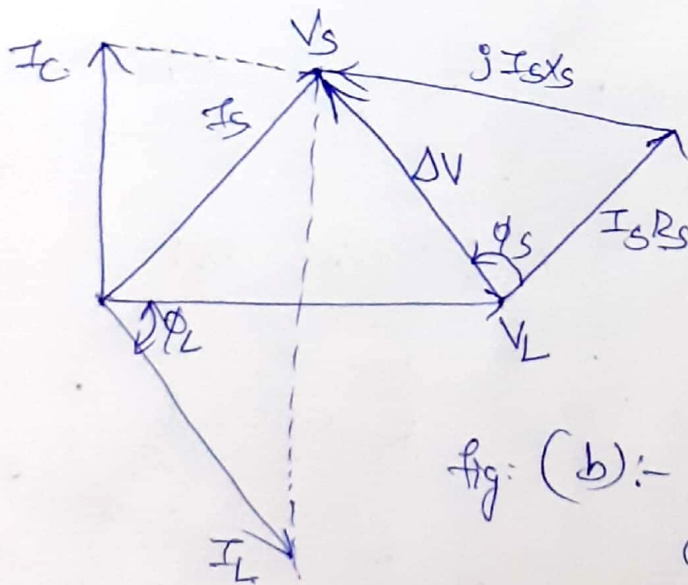


fig: (b):- phasor diagram for a compensated system



The supply reactive power with a compensator is

(15)

$$Q_s = Q_c + Q_L$$

$Q_c$  is adjusted in such a way that  $\Delta V = 0$

i.e.,  $|V_s| = |V_L|$

from eq (1) & eq (3) we get

$$|V_s^2| = \left[ |V_L| + \frac{R_s P_L + X_s Q_s}{|V_L|} \right]^2 + \left[ \frac{X_s P_L - R_s Q_s}{|V_L|} \right]^2$$

Simplifying and rearranging eq (d)

→ eq (4)

$$|V_s^2| = |V_L^2| + \left[ \frac{R_s P_L + X_s Q_s}{|V_L|} \right]^2 + \frac{2(R_s P_L + X_s Q_s) \cdot |V_L|}{|V_L|} + \frac{X_s^2 P_L^2 + R_s^2 Q_s^2 - 2X_s P_L R_s Q_s}{|V_L^2|}$$

$$= |V_L^2| + \frac{R_s^2 P_L^2 + X_s^2 Q_s^2 + 2R_s P_L Q_s X_s}{|V_L^2|} + 2(R_s P_L + X_s Q_s) + \frac{X_s^2 P_L^2 + R_s^2 Q_s^2 - 2X_s P_L R_s Q_s}{|V_L^2|}$$

$$= |V_L^2| + \frac{R_s^2 P_L^2 + X_s^2 Q_s^2 - 2X_s P_L R_s Q_s}{|V_L^2|} + \dots$$

$$\begin{aligned} |V_s^2 V_L^2| &= |V_L^4| + R_s^2 P_L^2 + X_s^2 Q_s^2 + 2(R_s P_L + X_s Q_s) |V_L^2| + X_s^2 P_L^2 + R_s^2 Q_s^2 \\ &= Q_s^2 (R_s^2 + X_s^2) + Q_s (2 |V_L^2| X_s) + |V_L^4| + P_L^2 (R_s^2 + X_s^2) + 2 R_s P_L |V_L^2| \end{aligned}$$

$$\therefore Q_s^2 (R_s^2 + X_s^2) + Q_s (2|V_L^2|X_s) + |V_L^4| + P_L^2 (R_s^2 + X_s^2) + 2R_s P_L |V_L^2| - |V_s^2 V_L^2| = 0$$

The above equation can be represented in a compact form as.

$$a Q_s^2 + b Q_s + c = 0$$

where,  $a = R_s^2 + X_s^2$

$$b = 2|V_L^2|X_s$$

$$c = (V_L^2 + R_s P_L)^2 + X_s^2 P_L^2 - |V_s^2 V_L^2|$$

$$Q_s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The value of 'Qc' is found using the above equation by using the compensator reactive power balance equation  $|V_s| = |V_L|$  &  $Q_c = Q_s - Q_L$

Here, the compensator can perform as an ideal voltage regulator, i.e., the magnitude of the voltage is being controlled, its phase varies continuously with the load current, where as the compensator acting as a P.F corrector reduces the reactive power supplied by the system to zero i.e.  $Q_s = 0 = Q_L + Q_c$

the following Equation can be reduced to

$$\Delta V = \frac{R_s P_L + X_s Q_L}{V_L} + j \frac{X_s P_L - R_s Q_L}{V_L} = \frac{R_s + j X_s P_L}{V_L}$$



$$\Delta V = (R_s + jX_s) \frac{P_L}{V_L}$$

(17)

So,  $\Delta V$  is independent of the load reactive power.

From this, we conclude that a pure reactive compensator cannot maintain both constant voltage & unity p.f. Simultaneously.

### SPECIFICATIONS OF LOAD COMPENSATION :-

The specifications of load compensation are:

1. The maximum & continuous reactive power requirement in terms of absorbing as well as generation.
2. Overload rating & duration.
3. Rated voltage & limits of voltage b/w which the reactive power rating must not exceeded.
4. Frequency and its variation.
5. Accuracy of voltage regulation requirement.
6. Special control requirement.
7. Maximum harmonic distortion with compensation in series.
8. Emergency procedure & precautions.
9. Response time of the compensator for a specified disturbance.
10. Reliability & redundancy of components.

## SERIES AND SHUNT COMPENSATION :-

The objective of series compensation is to cancel part of the series inductive reactance of the line using series capacitors, which results in the following factors

- i) Increase in maximum transferable power supply.
- ii) Decrease in transmission angle for considerable amount of power transfer.
- iii) Increase in virtual surge impedance loading

The effects of series and shunt compensation of overhead transmission lines are as follows:

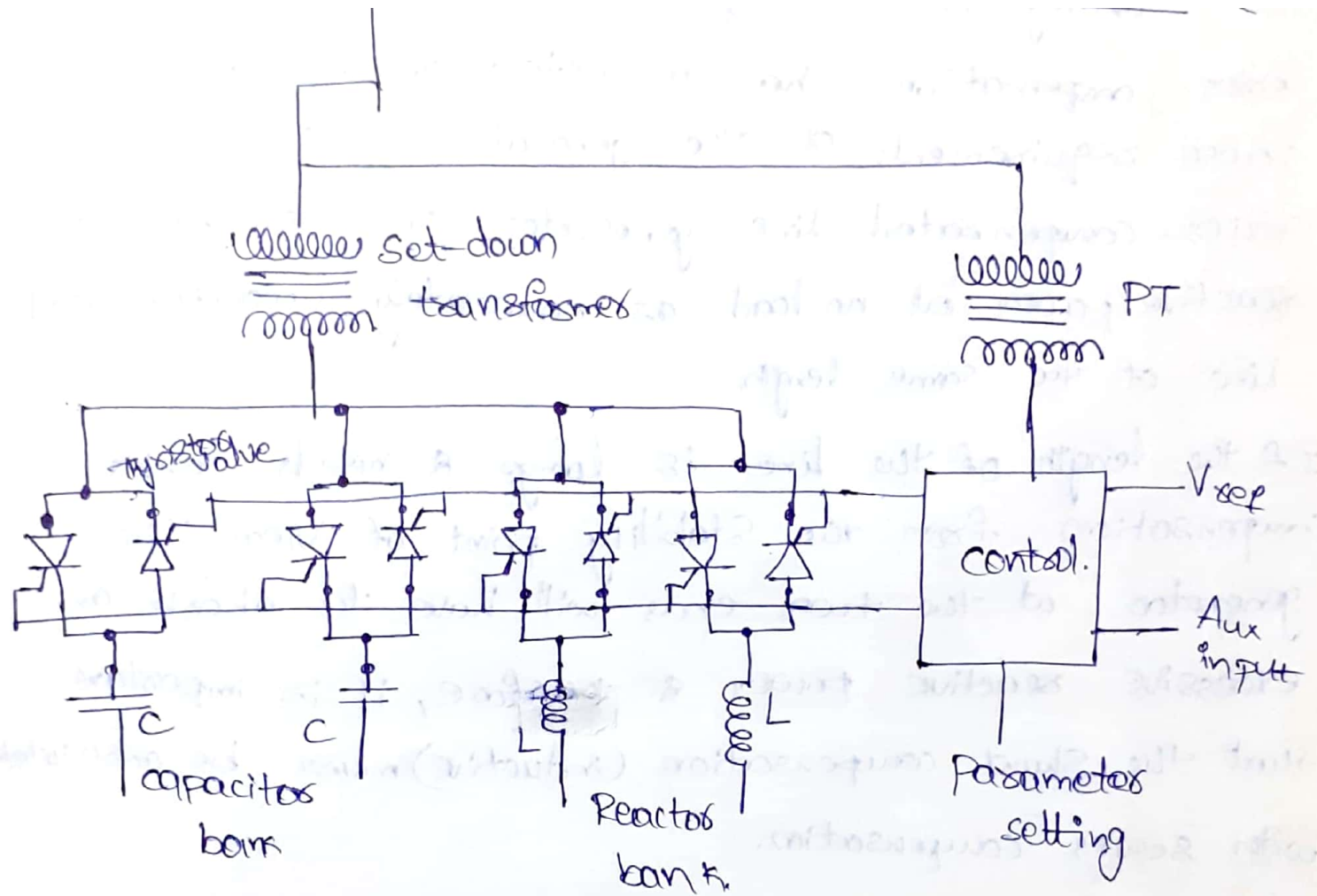
1. For a fixed degree of series compensation, the capacitive shunt compensation decreases the virtual surge impedance loading of the line. However, the inductive shunt compensation increases the virtual surge impedance & decreases the virtual surge impedance loading of the line.
2. If the inductive shunt compensation is 100%, then the virtual surge impedance becomes infinite & the loading is zero, which implies that a flat voltage profile exists at zero loads & the Ferranti effect can be eliminated by the use of shunt reactors.
3. Under a heavy-load condition, the flat voltage profile can be obtained by using shunt capacitors.
4. A flat voltage profile can be obtained by series compensation for heavy loading condition.



- 5. Voltage control using series capacitors is not recommended due to the lumped nature of series capacitors, but normally they are preferred for improving the stability of the system.
- 6. Series compensation has no effect on the load-reactive power requirements of the generator  $G$ , therefore, the series-compensated line generates as much line-charging reactive power at no load as completely uncompensated line of the same length.
- 7. If the length of the line is large & needs series compensation from the stability point of view, the generators at the two ends will have to absorb an excessive reactive power & therefore, it is important that the shunt compensation (inductive) must be associated with series compensation.

SHUNT COMPENSATOR:-

A shunt-connected static VAR compensator, composed of thyristor-switched capacitors (TSCs) and thyristor-controlled reactors (TCRs), is shown in fig(i) with proper co-ordination of the capacitor switching & reactor control, the VAR output can be varied continuously between the capacitive & inductive rating of the equipment. The compensator is normally operated to regulate the voltage of the transmission system at a selected terminal, often with an appropriate modulation



Fig(i) :- Static VAR compensator employing TCC and TCR.

i) Thyristor - controlled reactor :-

A shunt-connected thyristor-controlled inductor has an effective reactance, which is varied in a continuous manner by partial-conduction control of the thyristor valve.

With the increase in the size & complexity of a power system, fast reactive power compensation has



become necessary in order to maintain the stability (21) of the system. The thyristor-controlled shunt reactors have made it possible to reduce the response time to a few milliseconds. Thus, the reactive power compensators utilizing the thyristor-controlled shunt reactors become popular. An elementary single-phase TCR is shown in

fig (ii):-

It consists of a fixed reactor of inductance 'L' and a bidirectional thyristor valve. The thyristor valve can be brought into conduction by the application of a gate pulse to the thyristor, and the valve will be automatically blocked immediately after the AC current crosses zero.

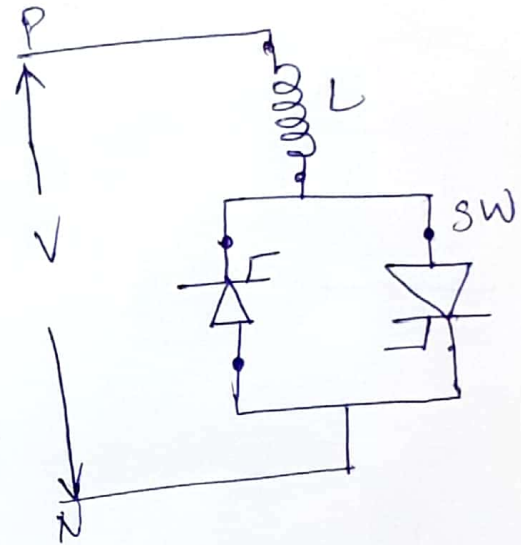


fig (ii):-

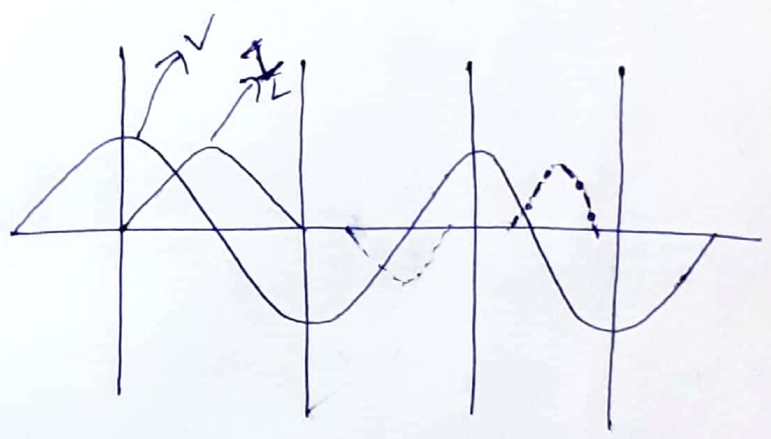
The current in the reactor can be controlled from maximum to zero by the method of firing angle delay. The effect of increasing the firing angle is to reduce the fundamental component of current. This is equivalent to an increase in the inductance of the reactor, reducing its current. As far as the fundamental component of current is concerned, the TCR is a controllable susceptance and can, therefore, be used as a static compensator. The current in this circuit is essentially

reactive, lagging the voltage by  $90^\circ$  and this is continuously controlled by the phase control of the thyristors. The conduction angle control results in a non-sinusoidal current wave form in the reactor.

In other words, the TCR generates harmonics. For identical positive and negative current half-cycle time, only odd harmonics are generated as shown in fig(iii). By using filters, we can reduce the magnitude of harmonics.

- voltage
- - -  $I_1 \cos \alpha_1$
- . -  $I_1 \cos \alpha_2$
- $I_1 \cos \alpha$

$\alpha = 90^\circ$   
 $\alpha < \alpha_1 < \alpha_2 < 180^\circ$



fig(iii) :- TCR wave forms

TCR's characteristics are:

- 1) continuous control
- 2) No transients
- 3) Generation of harmonics

2) THYRISTOR - SWITCHED CAPACITOR :-

Unlike shunt reactors, shunt capacitors cannot be switched into or out of the system individually. The control is done continuously by sensing the load Vars. A single-phase TSC is shown in fig(iv) :-



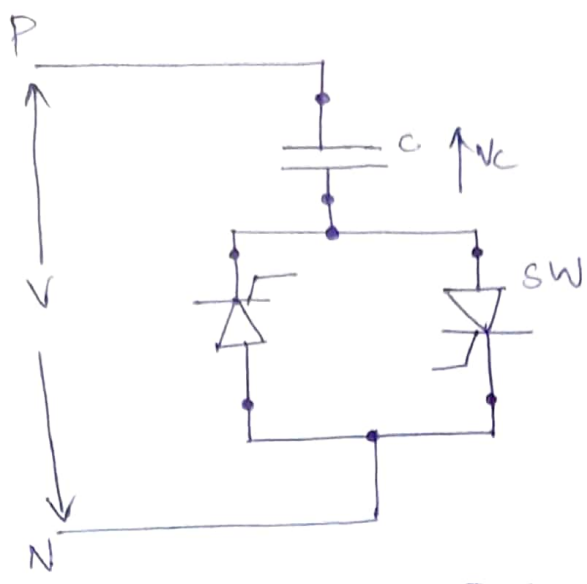


Fig (iv) :- TSC (Thyristor switched capacitor).

It consists of a capacitor, a bidirectional thyristor valve, and relatively small surge current in the thyristor valve under abnormal operating conditions (e.g. control mal-function causing capacitor switching at a 'wrong time'), it may also be used to avoid resonance with system impedance at particular frequencies.

The problem of achieving transient-free switching of the capacitor is overcome by keeping the capacitor charged to the '+ve' or '-ve' peak value of the fundamental frequency network voltage at all times when they are in the stand by state. The switching-on-transient is then selected at the time when the same polarity exists in the capacitor voltage.

This ensures that switching on takes place at the natural zero passage of the capacitor current. The switching thus takes place with practically no transients.

This is called zero-current switching.

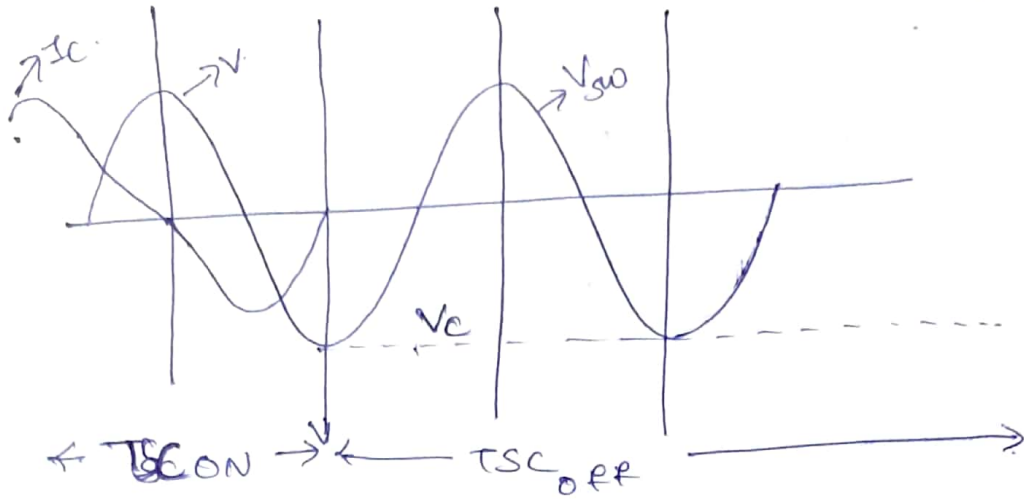


Fig (v) :- TSC wave forms.

Switching off a capacitor is accomplished by suppression-offering pulses to the anti-parallel thyristors so that the thyristors will switch-off as soon as the current becomes zero. In principle, the capacitor will then remain charged to the positive or negative peak voltage and be prepared for a new transient-free switching-on as shown in fig(v).

TSC's characteristics are:

- i) steeped control
- ii) No transients
- iii) No harmonics
- iv) Low losses
- v) Redundancy and flexibility.

### SERIES COMPENSATOR :-

In the TSC scheme, increasing the number of capacitor bank in series, controls the degree of series compensation. To accomplish this, each capacitor bank is controlled by a thyristor bypass switch or valve.



The operation of the thyristor switches is co-ordinated with voltage & current zero-crossing; the thyristor switch can be turned on to bypass the capacitor bank when the applied AC voltage crosses zero, and its turn-off has to be initiated prior to a current zero at which it can recover its voltage-blocking capability to activate the capacitor bank.

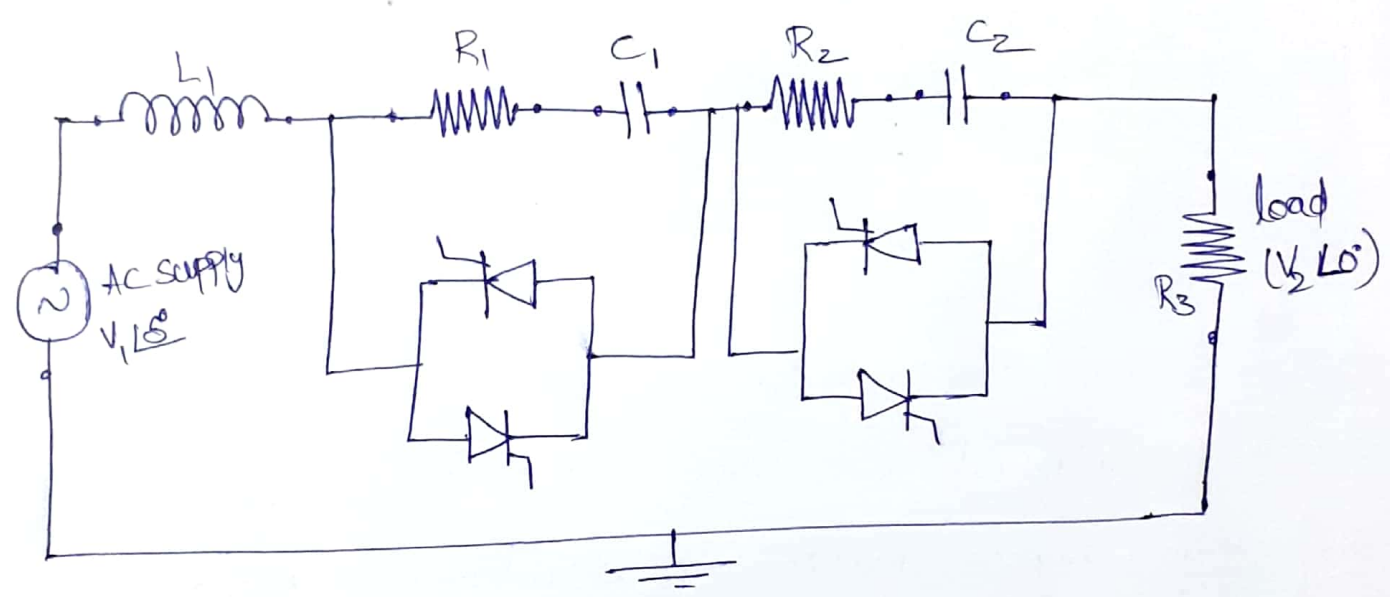


Fig (i) :- series compensator.

Initially, capacitor is charged to some voltage, while switching the SCR's, they may get damaged because of this initial voltage. In order to protect the SCR's from this kind of damage, resistor is connected in series with capacitor as shown in fig(i).

In a fixed capacitor, the TCR scheme as shown in fig(ii) & fig(iii), the degree of series compensation in the capacitive operating region is

increased (or decreased) by increasing (or decreasing) the current in the TCR.

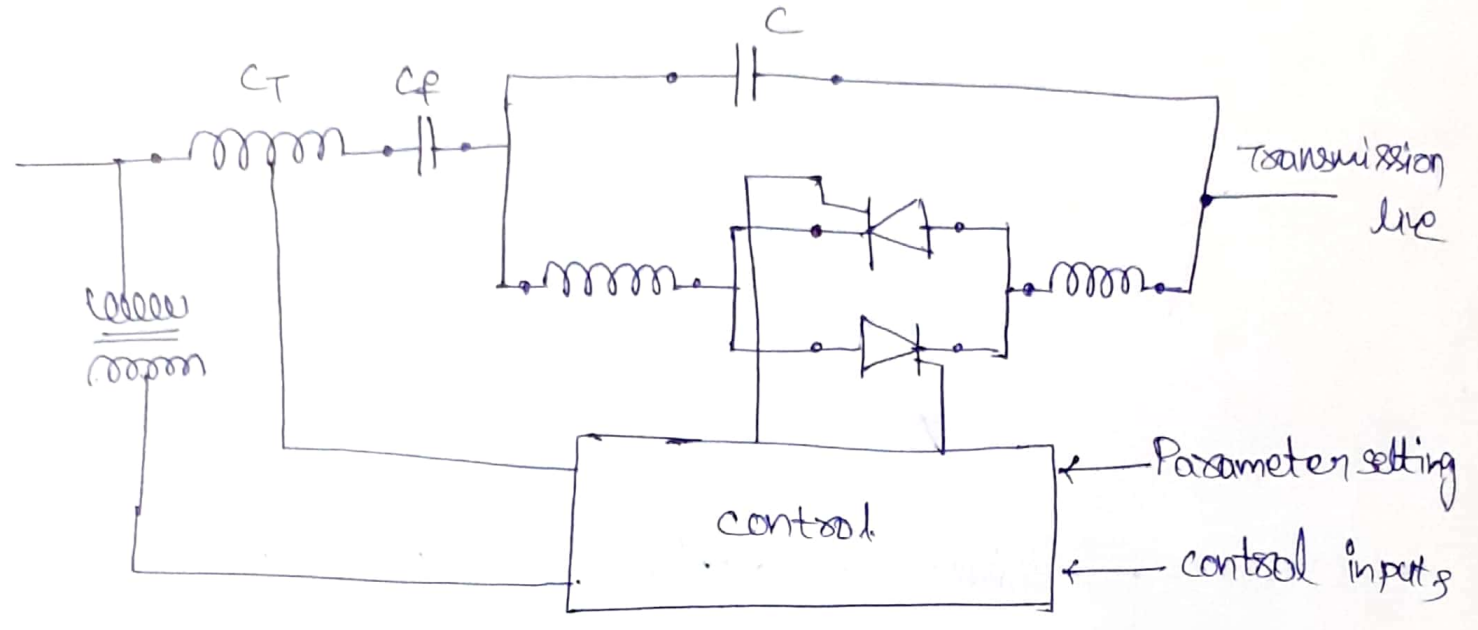


Fig (i): thyristor - controlled capacitor

Minimum series compensation is reached when the TCR is switched off. The TCR may be designed for substantially higher maximum admittance at full thyristor conduction than that of the fixed shunt-connected capacitor.

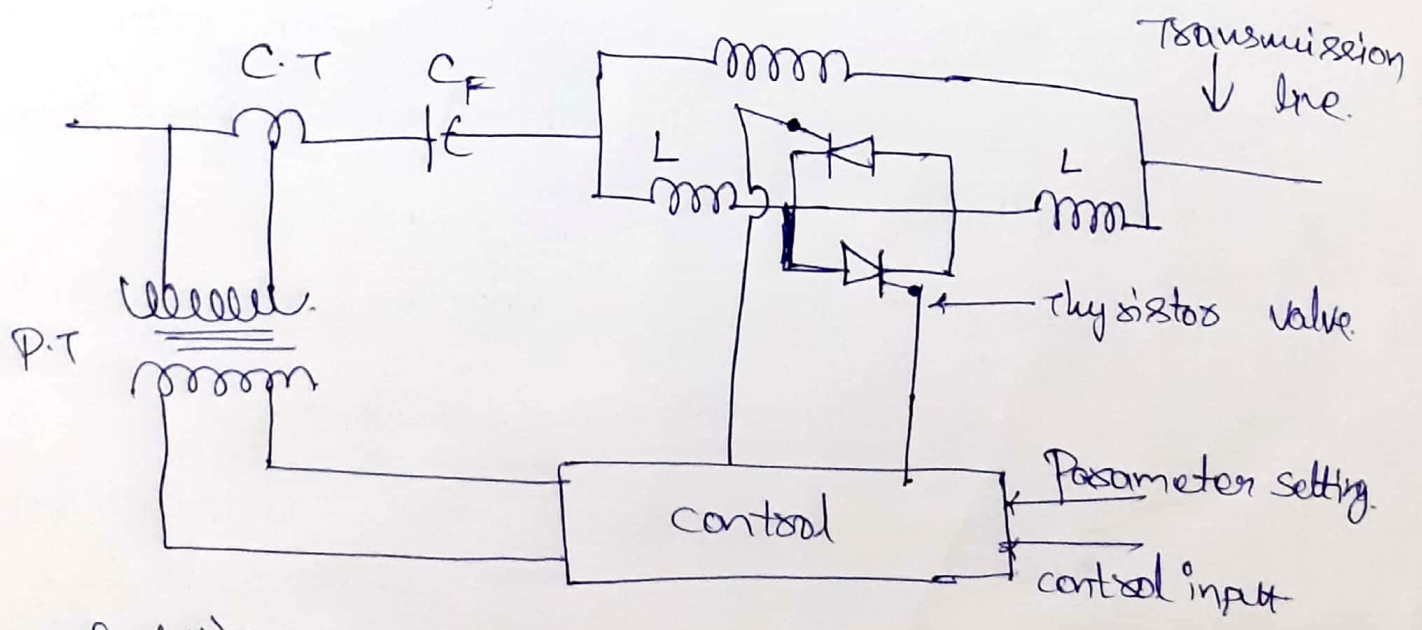


Fig (ii): TCR.



In this case, the TCR, time with an appropriate

Surge-current rating can be used essentially as a bypass switch to limit the voltage across the capacitor during faults and the system contingencies of similar effects.

controllable series compensation can be highly effective in damping power oscillation and preventing loop flows of power.

The expression for power transmitted is given by

$$P = \frac{V_s V_r}{X} \sin \delta$$

where,  $V_s$  = sending-end voltage

$V_r$  = receiving-end voltage

$\delta$  = Angle b/w  $V_s$  &  $V_r$

$X = X_L - X_C$

controlled series compensation is a useful means for optimizing power flow between regions for varying loading and network configurations. It becomes possible to control power flows in order to achieve a number of goals that are listed below:

- 1) Minimizing system losses
- 2) Reduction of loop flows.
- 3) Elimination of line overloads
- 4) Optimizing load sharing b/w parallel circuits
- 5) Directing power flows along contractual paths.

## Chapter 3

# Power System Operation in Competitive Environment

### 1. INTRODUCTION

In Chapter-1 we discussed the various factors and issues, that initiated the process of deregulation of the electric power industry all over the world. Subsequently in Chapter-2 the basic concepts of economic operation of the system were discussed from a classical perspective, *i.e.*, considering the utility to be operating in a vertically integrated environment. In this classical perspective, the system operator seeks to achieve economic efficiency for the system as a whole. The operating paradigm is based on achieving the system least-cost solution while meeting reliability and security requirements.

Given this background, we shall now examine the various issues related to the deregulated power industry and the new paradigms of system operation. First we shall discuss power sector deregulation in some more detail with emphasis on the two distinct structures of deregulation that have emerged. As we have mentioned earlier, several players have emerged in the deregulated electricity markets. Consequently, many of the activities of system operation have been taken over by different entities. For example, the genco, the transco and the system operator, each now have a role to play, independent of each other, while at the same time needing to coordinate their activities in order to maintain the system security and reliability.



## 2. ROLE OF THE INDEPENDENT SYSTEM OPERATOR (ISO)

The independent system operator (ISO) is the central entity to have emerged in all deregulated markets with the responsibility of ensuring system security and reliability, fair and equitable transmission tariffs, and providing for other system services. With differing market structures evolving in various countries, it has been noticed that based on the responsibilities assigned to them and their functional differences, ISOs could be placed in two categories.

The first and the more common one, is the pool structure in which the ISO is responsible for both market settlement including scheduling and dispatch, and transmission system management including transmission pricing, and security aspects. This has often been referred to, as the *pool model* and it exists in different forms in the UK, Australian, Latin American and some of the US markets.

The other structure is that of open access, one dominated by bilateral contracts, and can be found in the Nordic countries. In this system, bulk of the energy transactions are directly organized between the generator and the customer, and the ISO has no role in generation scheduling or dispatch and is only responsible for system operation. The role of the ISO is minimal and limited to the maintenance of system security and reliability functions. *Table 1* lists the basic differences between these two market structures and the role of the ISO in each.

*Table 1. Comparison of the Two Different Market Structures*

Open Access	Pool
<ul style="list-style-type: none"> <li>◆ Bulk of the energy transactions are carried out as bilateral trades while there may also exist a day-ahead spot market</li> <li>◆ The ISO is not responsible for market administration, generation scheduling or dispatch functions</li> <li>◆ Market administration is carried out by a separate entity and participation in the market is not mandatory</li> <li>◆ The ISO is responsible for system security and control, procuring necessary ancillary services</li> <li>◆ Example: Nordic markets</li> </ul>	<ul style="list-style-type: none"> <li>◆ All energy transactions are carried out through the pool which may be organized through a day-ahead trading mechanism</li> <li>◆ The ISO is responsible for the market settlements, unit commitment, and determination of the pool price</li> <li>◆ Market administration is carried out by the ISO and participation of generators is mandatory</li> <li>◆ The ISO is also responsible for system security and control, procuring necessary ancillary services</li> <li>◆ Example: UK market</li> </ul>

In any market structure, be it a pool or bilateral contract dominated one, the ISO has three basic functions laid out for it, maintenance of system security and reliability, service quality assurance and promotion of economic



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## Reasons for restructuring / deregulation of power industry

The next obvious question is, "what is deregulation or restructuring of an industry?" From the name, one can sense discontinuation of the framework provided by the regulation. In other words, deregulation is about removing control over the prices with introduction of market players in the sector. However, this is not correct in a strict sense. An overnight change in the power business framework with provision of entry to competing suppliers and subjecting prices to market interaction, would not work successfully. There are certain conditions that create a conducive environment for the competition to work. These conditions need to be satisfied while deregulating or restructuring a system. Sometimes, the word 'deregulation' may sound a misnomer. 'Deregulation' does not mean that the rules won't exist. The rules will still be there, however, a new framework would be created to operate the power industry. That is why the word 'deregulation' finds its substitutes like 're-regulation', 'reforms', 'restructuring', etc. The commonly used word in Europe is 'liberalization' of power industry; 'deregulation' is a more popular phrase in US.

If the power industries worked successfully with the regulated monopoly framework for over 100 years, what was the need for deregulating or changing the business framework of the system? There are many reasons that fuelled the concept of deregulation of the power industry. One major thought that prevailed during the early nineties raised questions about the performance of monopoly utilities. The takers of this thought advocated that monopoly status of the electric utilities did not provide any incentive for its efficient operation. In privately owned utilities, the costs incurred by the utility were directly imposed upon the consumers. In government linked public utilities, factors other than the economics, for example, treatment of all public utilities at par, overstaffing, etc. resulted in a sluggish performance of these utilities. The economists started promoting introduction of a competitive market for electrical energy as a means of benefit for the overall power sector. This argument was supported by the successful reform experiences of other sectors such as airlines, gas, telephone, etc.

Another impetus for deregulation of power industry was provided by the change in power generation technology. In the earlier days, cost-effective power generation was possible only with the help of mammoth thermal (coal/nuclear) plants. However, during the mid eighties, the gas turbines started generating cost effective power with smaller plant size. It was then possible to build the power plants near the load centers and also, an opportunity was created for private players to generate power and sell the same to the existing utility. This technology change, supposed to have provided acceleration to the concept of independent power producers, supported the concept of deregulation further. This technology change is supposed to have provided acceleration to the concept of independent power producers. This further supported concept of deregulation. This was specifically true where the financial losses were apparently high which was prevalent in some of the developing countries.

It should be noted that these are the indicative or major reasons for introducing the concept of deregulation in power industry. There are many other reasons as well. One



of the important reasons is the condition under which power systems were regulated, did not exist any more. There was no wind of skepticism about the electrical technology and all the initial investments in infrastructure were already paid back. Further, the deregulation aims at introducing competition at various levels of power industry. The competition is likely to bring down the cost of electricity. Then, the activities of the power industry would become customer centric.

The competitive environment offers a good range of benefits for the customers as well as the private entities. It is claimed that some of the significant benefits of power industry deregulation would include:

1. Electricity price will go down: It is a common understanding that the competitive prices are lesser than the monopolist prices. The producer will try to sell the power at its marginal cost, in a perfectly competitive environment.
2. Choice for customers: The customer will have choice for its retailer. The retailers will compete not only on the price offered but also on the other facilities provided to the customers. These could include better plans, better reliability, better quality, etc.
3. Customer-centric service: The retailers would provide better service than what the monopolist would do.
4. Innovation: The regulatory process and lack of competition gave electric utilities no incentive to improve or to take risks on new ideas that might increase the customer value. Under deregulated environment, the electric utility will always try to innovate something for the betterment of service and in turn save costs and maximize the profit.

The deregulation of the industry has provided electrical energy with a new dimension where it is being considered as a commodity. The 'commodity' status given to electrical power has attracted entry of private players in the sector. The private players make the whole business challenging from the system operator's point of view, as it now starts dealing with many players which are not under its direct control. This calls for introduction of fair and transparent set of rules for running the power business. The market design structure plays an important role in successful deregulation of power industry.

Like most lists of desiderata, the above requirements can only be met partially by any practical transmission-pricing scheme, which requires some tradeoffs and compromises. One of the basic tradeoffs in this context is between simplicity and short-term efficiency. The questions that one must address in making this tradeoff are:

- How precise need the short-term economic signals be to move the system toward long-term efficiency?
- What is the economic cost of a simpler and less accurate pricing scheme?
- What is the correct level of precision in short term price signals given the approximations in the system models, the formulation of objectives and the available computation technology?
- How we compare the economic value of an accurate ex-post price signal to an approximate ex-ante price signal?

Another important tradeoff concerns the desired level of decentralization. The two basic paradigms for optimal resource allocation are the central planning approach versus the decentralized "invisible hand" approach. The equivalence of the results in an idealized theoretical setting has enabled economists to simulate market outcomes by using optimization models. Indeed market simulation is the proper use of optimization technology in a competitive market setting. The endlessly debated philosophical question is to what extent should a market simulation model be used to manage the market. This debate dates back to the 1920's when the concept of planned economies was first proposed. Using a market simulation model to set prices as advocated by some in the electricity restructuring debate is somewhat analogous to using statistical polling to determine (rather than just forecast) election results. The physical characteristics of electricity and the network aspects of electricity transmission justify a certain degree of central coordination in order to maintain system reliability. However, the extent to which central intervention based on optimization models should be used to insure short-term economic efficiency is a debatable policy decision with direct consequences for transmission pricing and congestion management protocols. The argument for a centralized market management approach is that given the need for central coordination (for reliability considerations) it would be foolish not to take the extra step and optimize resource use so as to maximize short term efficiency. The counter argument is that, multiperiod efficiency need not collapse to the sum of single period efficiencies. Massive integration of systems on cost-based efficiency grounds can come at a loss of gains from competition and innovation born of profits. Decentralization and minimal central intervention in the market will promote long term efficiency by facilitating interaction between buyers and seller, customer choice, intermediation and technological innovation. The basic question is then does the long term potential gain justify the short term losses caused by the response lags and inefficiencies of a decentralized system.

The differences among the various proposed schemes for definition of transmission rights, transmission pricing and congestion management can be categorized along several dimensions as follows:

- Physical vs. financial transmission rights
- Link based vs. node based (point to point) definition of transmission rights
- Access based pricing vs. usage based pricing
- Locational differentiation in tariffs: nodes, zones or uniform prices
- Ex-ante vs. ex-post pricing
- Bundling of transmission service and energy vs. treating energy and transmission service as separate commodities
- Congestion management through efficient generation dispatch vs. efficient congestion relief.

It is beyond the scope of this paper to provide a comprehensive survey of all the proposed approaches and even if we tried we would probably miss some. Instead we will focus on two basic ideas related to the simplification and decentralization of decision making in the deregulated electric power industry. First I will explain how the California congestion management approach has been able to separate the energy market from the transmission market by, effectively, treating congestion relief as an ancillary service. Next I will discuss the issue of zonal aggregation and describe a new priority zonal network access pricing approach that may be viewed as an extension of the familiar postage stamp approach. The proposed scheme enables efficient intrazonal congestion management based on a relatively simple ex-ante transmission tariff. In order to put these ideas in context I will first contrast the two extreme approaches on the simplicity vs. efficiency tradeoffs.



## Simplicity vs. Efficiency: Is Nodal Pricing Worth the Trouble?

Two opposite extremes in terms of the tradeoff between short-term efficiency and simplicity in transmission pricing are the nodal pricing approach and the postage stamp approach. In the latter transmission pricing takes the form of a fixed ex-ante charge per MWh for transmission service between any two points in the grid. The simplicity and certainty of this approach is compelling from the point of view of energy trading over the grid. However, it has been argued that the lack of locational differentiation results in no economic signals to investors and users for efficient location of new load (e.g. production facilities) and for the location of new generation and transmission lines. Furthermore, postage stamp transmission pricing does not elicit economic signals from customers that could be used to manage congestion efficiently. There is, however, little evidence as to the magnitude of the efficiency losses resulting from the lack of correct economic signals and the debate is raging with regard to how precise these signals need be to recapture most of these losses.

Motivated by short-run efficiency considerations, the nodal pricing approach advocated by Hogan [1992] manages congestion and sets transmission prices through a centralized energy market based on economic dispatch. The basic idea of the nodal pricing approach is to organize the market as a pool in which generators (and ideally loads) submit hourly bids for node specific injection and withdrawals of power to an Independent System Operator (ISO) with full coordination and price setting authority. The ISO minimizes the total system's gain from trade (demand bids less supply bids) subject to transmission and reliability constraints. The price at each node is then set to the incremental bid price of the most expensive unit generated or consumed at that node. These nodal prices become the hourly prices charged to loads and paid to generators at the respective nodes. When there is no congestion all nodal prices are in theory identical. However, even congestion on a single link could result in a different price at every node in the system (in the WSCC there are around 2500 such nodes).

Proponents of the nodal pricing approach claim that bilateral transactions can be readily accommodated within this framework. A physical bilateral transaction can be scheduled as if the injection submitted a zero bid and the load submitted an infinite bid. Such a transaction is then subject to an ex-post transmission charge that equal the opportunity cost of the transaction, i.e., the cost difference of selling the power to the pool at the injection node price and buying it back at the withdrawal node price. Thus, the transmission charge between any pair of nodes is set ex-post to the nodal price difference between the nodes. The cost off transmission, therefore, varies between each pair of locations and is only known to energy traders after the fact. Bilateral traders that wish to protect themselves against transmission price risk among two specific locations can do so in two ways. They can acquire transmission congestion contracts (TCCs) between the two locations. These are financial instruments underwritten by the ISO that entitles or (obligates) their holder to a payment that equals the nodal price difference between the nodes. Such a financial contract would enable a trader to fully hedge the transmission price risk between two nodes. Unfortunately, with prices being different at each node and the large number of different TCCs needed to enable full hedging for each possible bilateral transaction (a 3000 node system would require about 4.5 million of different TCCs) it is unlikely that a market for TCCs could achieve sufficient liquidity to make TCC pricing efficient. Without a liquid TCC market the value of these instruments as risk management tools for energy traders is questionable.

Bilateral traders can also manage transmission price risk by actively participating in congestion relief. The nodal pricing paradigm (as implemented for instance in the PJM pool) restricts such participation to incremental and decremental bids that can be readily interpreted within the framework of the central pool economic dispatch protocol. Specifically, a trader may submit incremental and decremental (inc/dec) bids that would allow the ISO to modify its injection as if it was a pool bidder. With demand side bidding it would also be possible to have inc/dec bids on the load side. Such inc/dec bids, however, allow the ISO to displace the bilateral generator by cheaper generation, for efficiency reasons, even if there is no congestion.

While proponents of nodal pricing based pools often argue that such systems (e.g. PJM or NYPP) can accommodate bilateral trading the reality is that only bilateral trading that fits within the rigid pool framework are allowed. It is not possible for a bilateral trader to limit its congestion risk without exposing its generators to "efficiency motivated displacement". A bilateral trader cannot submit an inc/dec bid and ask that it only be used if there is congestion. It is also not possible for a trader to cap its transmission cost by submitting a decremental bid or cap on the cost of transmission between two points (i.e. submit an inc/dec bid on the nodal price difference rather than on the nodal prices). Such a transaction cannot be decomposed into elementary sell/buy transactions within the pool and are, therefore, disallowed. Finally, as



it was painfully realized when the PJM pool moved to nodal pricing, the popular user choice contracts, where the buyer can take delivery at any chosen location, cannot be protected against transmission price risk. Proponents of nodal pricing dismiss the disallowed bilateral transactions on the grounds that such transactions are socially inefficient and no one in his right mind would prefer them to the superior deals allowed by the pool. For instance: why wouldn't a bilateral generator submitting a decremental bid for congestion relief be willing to accept replacement of its generation by power priced below its declared decremental price? Such arguments expose the central planning mentality underlying the nodal price approach which does not tolerate bilateral trading that cannot be rationalized within that limited short-term efficiency perspective.

The complexity and restrictive characteristics of the nodal pricing framework are rationalized on the grounds of short-term economic efficiency. However, such efficiency claims hinge on unrealistic or simplistic assumptions. In particular, efficient resource use under a nodal pricing scheme requires that each generator and load bid their true costs or willingness to pay. This is a highly questionable premise given the inevitability of some locational market power in the electricity industry. Furthermore, intertemporal costs and constraints that are accounted for in unit commitment optimization affect optimal dispatch in a broad sense. The power flow optimization that is used to determine nodal prices is predicated on unit commitment decision. However, central unit commitment optimization is fundamentally incompatible with a competitive market (see Johnson, Oren and Svoboda [1997]). It requires information about costs and constraints that is either not revealed to the ISO or is subject to gaming (like in the UK system). None of the proposals or current implementations of nodal pricing offers a credible way of assuring optimal unit commitment. Indeed many deregulated electricity system (e.g. Norway, Victoria pool, California) opted for self-commitment where the decision to turn a unit on or off is left to the individual generators and not centrally optimized.

In drawing conclusions from the above discussion I like to point out an important distinction between precision and accuracy. While nodal pricing aims at providing precise locational economic signals their accuracy is questionable and hardly justifies the complexity and rigidity of that approach. The ability of this approach to meet its idealized efficiency objectives is hindered by unrealistic premises and by the fact that it ignores intertemporal considerations. Furthermore, the importance of accurate short-term economic signals toward achieving long-term efficiency is debatable. All this suggests that a less radical approaches that are suboptimal (second best) in terms of theoretical short-term efficiency but simpler and more transparent would be more desirable than the nodal pricing alternative.

### **Efficient Congestion Relief without Mandatory Economic Dispatch**

Zonal pricing in conjunction with incremental and decremental (inc/dec) bids for congestion relief has been adopted in California as a less intrusive and simpler alternative to nodal pricing, which allows efficient congestion relief with minimal interference in the energy market. The basic principle of the California approach is to separate the energy market from congestion relief, which can be viewed as an ancillary service. Such separation empowers the ISO to use incremental and decremental bids by bilateral traders for the purpose of least cost congestion relief. The marginal congestion relief cost is then imposed on the serviced interzonal transactions as a congestion charge. The ISO, however, is not allowed to interfere in the energy market by using the inc/dec bids for the purpose of displacing "inefficient" generation beyond congestion relief needs.

The technical details of the California implementation are described by Papalexopoulos, Singh and Angelidis [1998]. I will illustrate here the basic principles of least cost congestion relief with a simple two-zone example. Figure 1 illustrates three interzonal bilateral transactions scheduled by three scheduling coordinators (SC). Each SC has its own supply and demand curve representing its load and generation capacity, which leads to its preferred schedule. In our example the preferred transaction quantity of each SC is chosen to maximize gains from trade (represented by shaded area). The corresponding cost of the marginal transaction unit varies across SCs (it is 17 for SC1, 15 for SC2 and 12 for SC3). It should also be noted that the energy settlement price of each SC depends on contractual arrangement and may differ from marginal cost. Each SC submits to the ISO a preferred schedule, which in this example is 1000MW. Since the interzonal transmission capacity is limited to 1500MW ISO intervention is needed to relieve congestion. In addition to a preferred schedule each SC submits inc/dec bids for congestion relief on both sides of the transmission line. The ISO's congestion relief protocol requires that the balanced schedule of



each SC be preserved (an adjustment for losses is made but we assume no losses in our example). Hence the inc/dec bids of each SC on the two side of the congested interface can be added to provide a congestion relief bid or supply function. In our example we assume that the congestion relief supply function of each SC reflects the opportunity cost (or forgone trade gains) due to backing down the transaction.

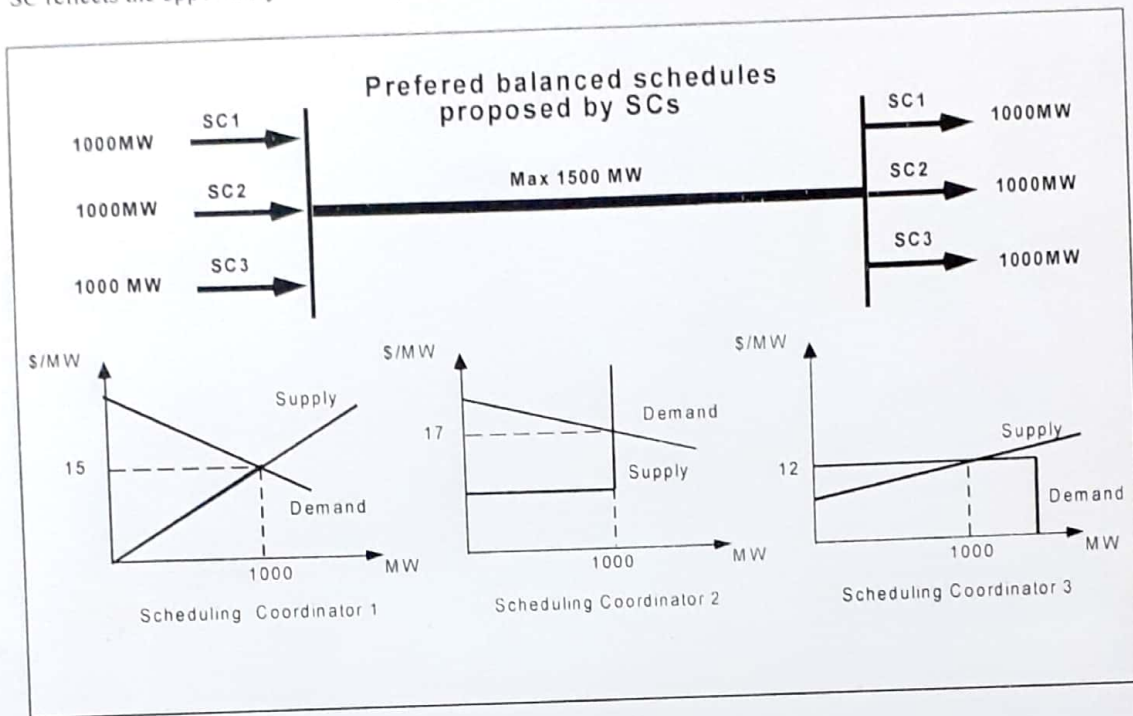


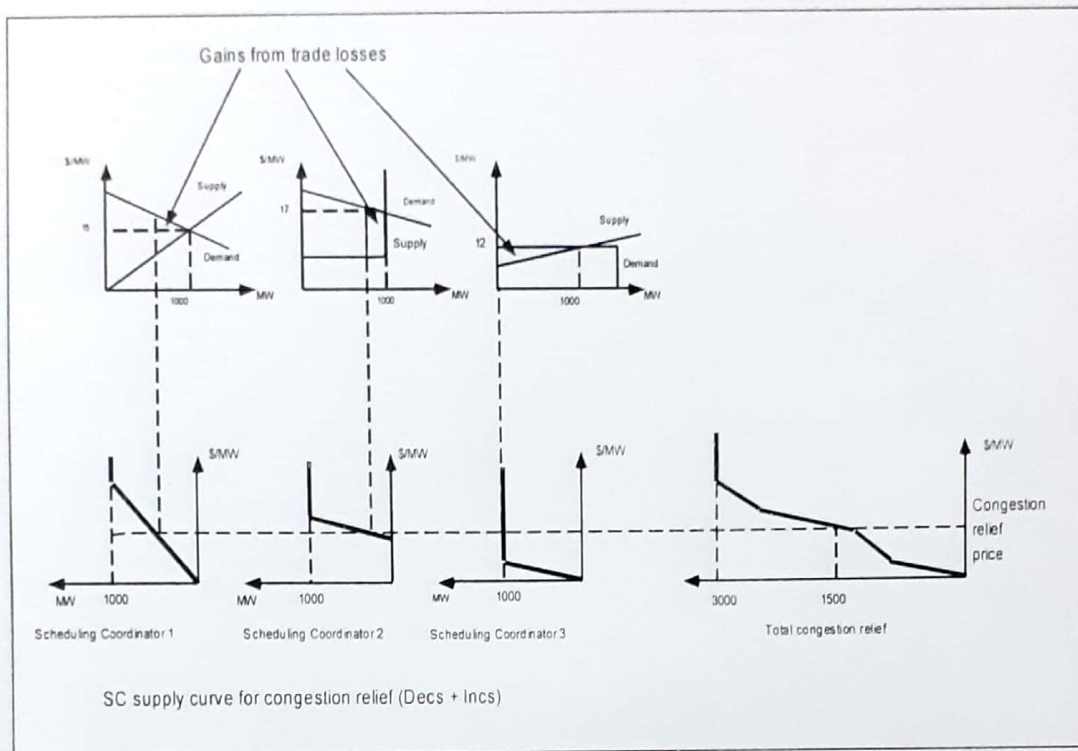
Figure 1

### The Preferred Schedules of Three Scheduling Coordinators Exceed Transmission Capacity

Figure 2 illustrates how the congestion relief supply function are ideally determined by each scheduling coordinator and how they are aggregated by the ISO into an overall congestion relief supply function for the congested interface. Using the aggregate congestion relief supply function the ISO determines the marginal congestion relief price that will back off transactions across the interface to a total of 1500 MW (the interface limit). That marginal price is used to determine the curtailment level for each scheduling coordinator according to their individual congestion relief supply curves. The surviving transactions are charged a transmission congestion fee (per MWh) that equals the congestion relief marginal clearing price. One of the implications of the above approach is that we view congestion relief as a commodity or more precisely an ancillary service. It is also efficient and meets comparability criteria since the marginal congestion relief opportunity cost is equalized across all curtailed transactions. What this approach does not do is equalize marginal cost of generation at a node across scheduling coordinators (as economic dispatch would do). In other words, congestion is relieved with minimal intervention in the energy market.

Figure 3 shows the injections and withdrawals of the three scheduling coordinators after the congestion relief protocol has been enforced along with the supply and demand curve of each scheduling coordinator (those represent private information that is not disclosed to the ISO). We observe that SC2 who is still generating 800 MW has a marginal cost that is higher than some of the curtailed generation capacity of SC3. Note, however, that SC2 was lucky enough to find a buyer that is willing to pay more and hence had a higher opportunity cost for curtailment. Should they desire to do so, SC2 could make a deal with SC3 to buy up to 600MW and turn its own generation down to 200MW. Since SC3 can produce these 600MW cheaper than SC2 both scheduling coordinators could benefit from such a transaction. The adjusted injections after such a transaction are shown at the bottom of Figure 3. Note, however, that such trading among the scheduling coordinators is voluntary and since it does not involve using the transmission lines it

does not involve the ISO. By contrast, in a nodal pricing approach, the ISO would have the authority and responsibility, to displace SC2's 600 MW with SC3 generation should the generators choose to participate in congestion relief by submitting decremental bids on the injection side.



**Figure 2**

**Scheduling Coordinators Submit Congestion Relief Bids Based on Their Opportunity Cost**

**Zonal Aggregation and Priority Access Pricing**

Zonal aggregation has been adapted in many systems as a realistic alternative to nodal prices. The basic idea in this approach is to divide the grid into few congestion zones as illustrated in Figure 4, with separate spot markets whose respective market clearing prices set the uniform price within the zone. In California, for instance, two such zones have been designated for the purpose of pricing power on the demand side but on the supply side a finer resolution is used. When there is no congestion the zonal markets collapse into one. However, when congestion is present the zonal markets are decoupled and the zonal market clearing prices reflect the supply and demand conditions in each zone as well as the interzonal transmission capability. When interzonal congestion occurs, bilateral transactions across zones are subject to an ex-post congestion fee based on the congestion relief cost between the zone. Keeping the number of distinct zones small facilitates the formation of liquid markets for zonal futures and forwards which enable traders to hedge the interzonal congestion cost risk. In California, for instance, the price difference between COB and Palo Verde electricity futures reflects the basis risk for congestion between north and south. Thus trader can use these financial instruments to hedge interzonal transmission cost risk between northern and southern California. The protocol for interzonal congestion relief and determination of the congestion charge can be accomplished through the congestion relief bidding system described in the previous section.



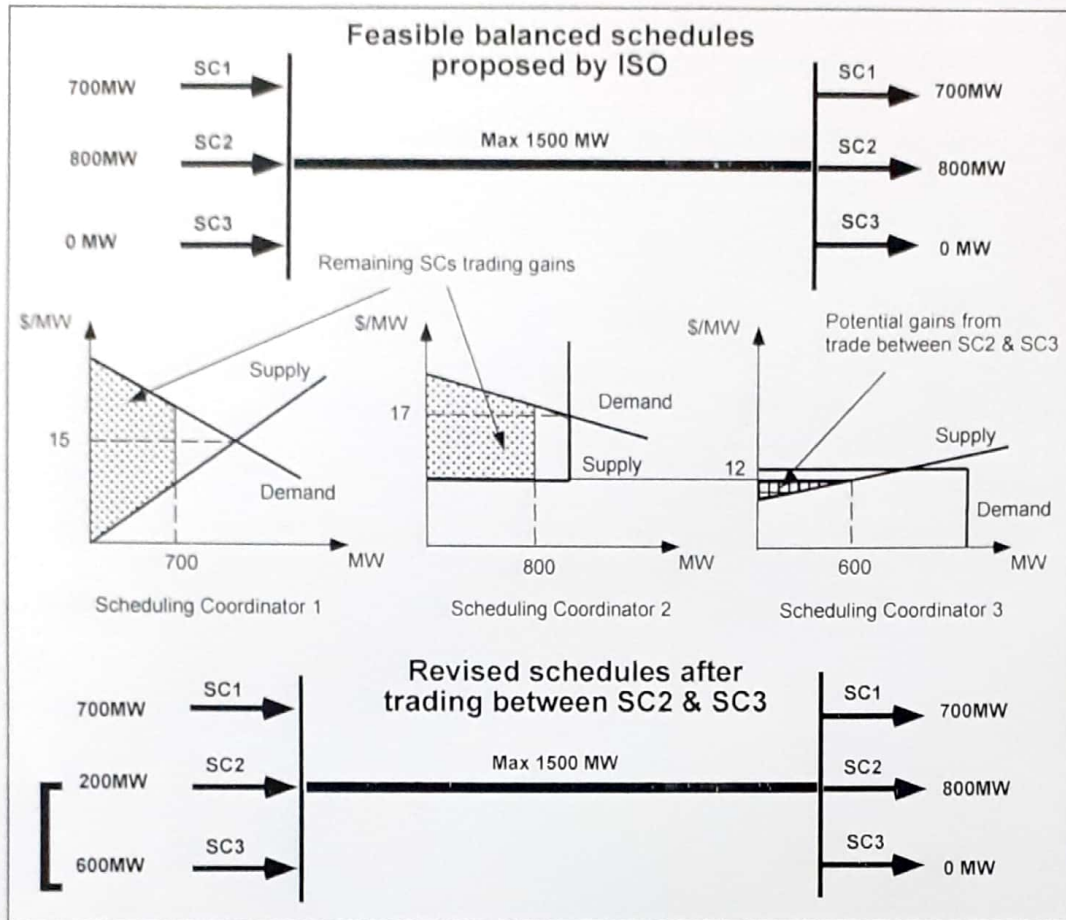


Figure 3

### Least Cost Congestion Relief and Energy Trading among Scheduling Coordinators

The basic question is how many zones are needed? Hogan [1997] argues that node aggregation can only be justified when nodal prices are identical at the grouped nodes and in order to know when that condition is met one must compute nodal prices anyway. On the other hand, Green [1998] shows using stylized data from the UK that the efficiency losses due to zonal aggregation are relatively small. The main issues that must be addressed when the number of zones is small (relative to the size and complexity of the network), are intrazonal congestion management and the provision of locational economic signals within the zone. In recent work Deng and Oren [1998] propose a new approach for addressing these issues by offering a uniform ex-ante priority differentiated zonal network access tariff in conjunction with an ex-post interzonal congestion fee. The proposed zonal tariff works as a zonal postage stamp method based on the injection zone, i.e., it is uniform regardless of the specific injection node within the zone. However, traders have the option to select among a variety of "stamps" which will determine their priority (place in line) should congestion occur (e.g. first class stamp, second class etc). While the stamp prices are uniform within the entire zone the actual probability of service they entail will depend on the location of the transaction. Consequently, traders can choose how much they wish to pay for transmission access by trading off the cost of the service priority against the locational risk of curtailment and their opportunity cost associated with curtailment. It is reasonable to expect that a high valued transaction or a transaction that impacts congestion prone links will opt for a higher priority level to reduce curtailment risk. On the other hand a transaction that is unlikely to be curtailed due to its location will opt for the least cost service. The ability of

## 6 Intra-Zonal Congestion

### 6.1 Introduction/Background

Scheduling Coordinators (SCs) submit day-ahead/hour-ahead generation and load schedules to the CAISO. Due to differences in the price and availability of power in different locations, these schedules vary daily and, collectively, may exceed the transfer capability of grid facilities within the congestion zones. However, the CAISO's Day Ahead and Hour Ahead Congestion Management Markets only manage congestion between zones, not within zones. This allows SCs to submit day-ahead/hour-ahead schedules that require transmission within a zone that is not physically feasible, and, as a consequence, creates the need for CAISO operators to have to manage intra-zonal congestion in real-time. Managing large amounts of intra-zonal congestion in real-time creates operational and reliability challenges and can result in significant costs.

Intra-zonal congestion costs are comprised of three components:

- 1) Minimum Load Cost Compensation (MLCC).<sup>1</sup> These costs result from generating units that are committed to operate on a day-ahead basis under the provisions of the Must-Offer Obligation in order to mitigate anticipated intra-zonal congestion.<sup>2</sup>
- 2) Costs from Reliability Must Run (RMR) real-time dispatches that are the first response to intra-zonal congestion.
- 3) Costs of Out-of-Sequence (OOS) dispatches.

Intra-zonal congestion most frequently occurs in load pockets, or areas where load is concentrated, where transmission within the zone is not sufficient to allow access to competitively-priced energy. In some cases, the CAISO must also decrement generation outside the load pocket to balance the incremental generation dispatched within it. Intra-zonal congestion can also occur due to pockets in which generation is clustered together, without the transmission necessary for the energy to flow out of that pocket to load. In both cases, the absence of sufficient transmission access to an area means that the CAISO has to resolve the problem locally, either by incrementing generation within a load pocket or by decrementing it in a generation pocket. Such congestion is inefficient if, over the course of congestion in that area, the market costs due to the transmission congestion (i.e., the cost imposed by the fact that load cannot be served by the lowest-cost supplier(s), and instead must be served by higher-cost suppliers) exceed the cost of a transmission upgrade that could alleviate the congestion.

Typically, there is limited competition within load or generation pockets, since the bulk of generation within such pockets is owned by just one or two suppliers. As a result, intra-zonal

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<sup>1</sup> MLCC payments are cost-based and are calculated as variable cost for providing the minimum load energy plus a \$6/MWh O&M adder.

<sup>2</sup> Pursuant to Amendment 60, MLCC costs are categorized into three categories (system, zonal and local), which reflect the primary reason the unit was denied a must-offer waiver. Both zonal and local MLCC costs are included as the MLCC component of intra-zonal costs.



pricing and congestion management without substantial short-term efficiency losses. Such simplifications offer more certainty of transmission cost to traders and better risk management capability. They also enable a higher degree of decentralization in the energy market, which will promote innovation and long-term efficiency.

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# Transmission Pricing and Congestion Management: Efficiency, Simplicity and Open Access

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## Abstract

Transmission pricing and congestion management are the key elements of a competitive electricity market based on direct access. They have also been the focus of much of the debate concerning alternative approaches to the market design and the implementation of a common carrier electricity system. This paper focuses on the tradeoffs between simplicity and economic efficiency in meeting the objectives of a transmission pricing and congestion management scheme. I contrast two extreme approaches: the postage stamp approach vs. nodal pricing. The paper questions the wisdom of the nodal pricing paradigm on the grounds of its rigidity and complexity. I argue that the theoretical efficiency properties of nodal pricing are unrealistic and do not justify the implementation drawbacks of the approach. The paper explains the underlying principles of least cost congestion relief, adopted in California that treat congestion relief as an ancillary service and enables the ISO to relieve congestion efficiently with minimal intervention in the energy market. I also discuss zonal aggregation and describe a new zonal priority network access pricing that complements interzonal congestion pricing by offering a market mechanism to guide intrazonal congestion management and provide economic signals for location of generation resources.

## Introduction:

There is general agreement among academics practitioners and policy makers that direct access to the transmission grid is the essential centerpiece for a competitive electricity market. Order 888 and Order 889 of the Federal Regulatory Energy Commission (FERC) reflect the role of direct access as the foundation for the electric power industry restructuring. These orders provide guidelines for nondiscriminatory transmission pricing and mandate timely disclosure of available transmission capacity but do not prescribe a particular approach to the institution of direct access. However, the prevailing restructuring paradigm being adopted in many states in the US has two key features: functional unbundling of generation transmission and distribution and the transfer of control over the transmission system to an Independent System Operator (ISO). The establishment of the ISO as a key institution in the emerging competitive electricity markets is based on the consensus that the physical characteristics of electricity impose requirements for real time central coordination in order to assure reliable service. However, the extent of centralized control and "market management" that is needed to assure system reliability and that is desirable from a social efficiency perspective has been a subject of public debate. That debate has polarized the restructuring approaches adopted so far on the east and west coasts of the US. This divergence manifests itself in the transmission pricing and congestion management protocols the centerpieces of a direct access system and two of the key functions of the ISO.

In an open access, competitive electricity system a transmission pricing scheme should fulfill several functions and meet various criteria:

- Generate revenues to compensate the owners of transmission assets
- Produce economic signals for efficient rationing of scarce transmission resources
- Produce economic signals for efficient investment in transmission and for efficient location of new generation capacity and loads
- Be simple to implement, transparent and conducive to energy trading.

→ non-judicial  
distribution  
costs  
of available  
things.  
typical  
example.



traders to self-select their service priority levels results in correct economic signal (direction wise) for efficient rationing when transmission capacity is scarce. It also produces the correct signal for location of new generation assets, since locating such assets where they do not impact congestion will allow their owners to save on transmission cost by selecting a lower service priority.

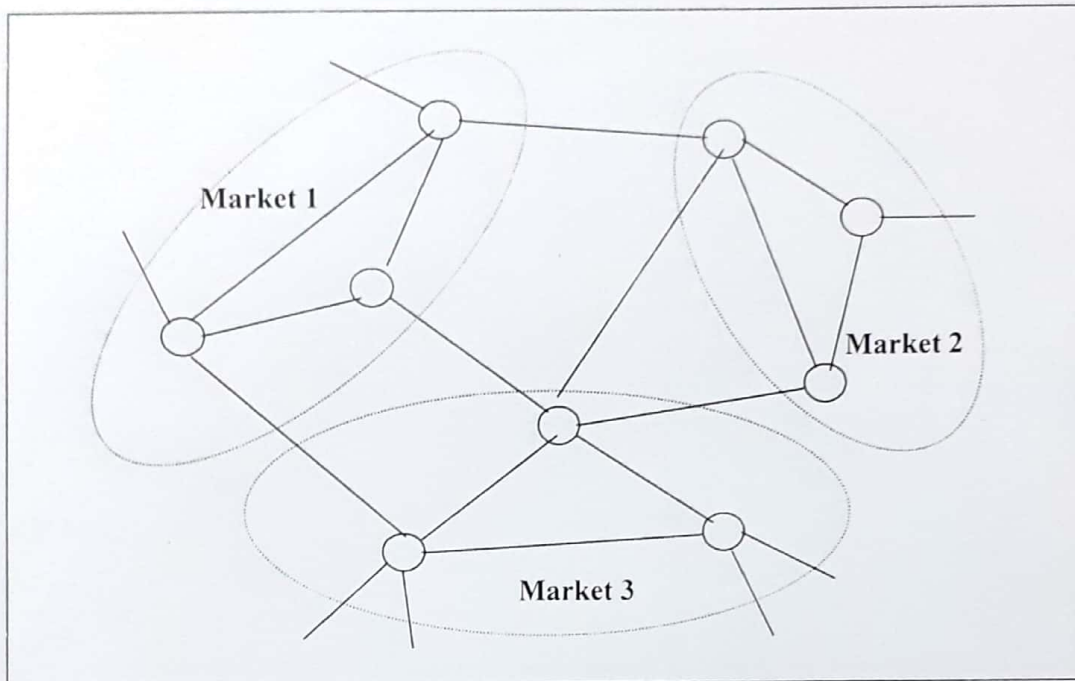


Figure 4

#### Zonal Aggregation of Nodes

There are many possible ways of implementing a priority network access tariff. The Pricing scheme proposed by Deng and Oren [1998] takes the form of an "option insurance". The basic premise of the proposed scheme is that the spot prices in each zone serve as a reference for replacement of curtailed delivery into the zone or for financial settlement for unfulfilled delivery contracts. So for example, if a trader holding a contract for delivery of power from zone 1 to zone 2 is curtailed his cost per curtailed MWh is the settlement or replacement cost in zone 2 ( $S_2$ ) less his saved marginal cost of generation ( $v$ ). In addition, we assume that an interzonal transaction is subject to an ex-post congestion fee per MWh that equals the difference of zonal spot prices ( $S_2 - S_1$ ). That fee is avoided when the transaction is curtailed. Hence, the trader incurs a net curtailment cost per MWh of  $(S_2 - v) - (S_2 - S_1) = S_1 - v$ . It follows that the opportunity cost of a curtailed transaction amounts to the opportunity cost of selling the power into the zonal spot market where the generator is located. Since the generator has the option not to generate when the spot market price is below his marginal cost, the opportunity cost of a generator in zone 1 of not having physical access to the network is  $\text{Min}[0, S_1 - v]$ . This opportunity cost is the same as the payoff of a call option with strike price  $v$ , with respect to the zonal spot price. Given the uncertainty in spot prices, the forward value of network access to a generator in zone 1 with marginal cost  $v$  is given by the actuarial value of a call option with strike price  $v$ . Figure 5 illustrates the fact that the value to a trader of transmission access is the same whether the transaction is interzonal or intrazonal due to the ex-post congestion fee imposed on interzonal transactions.

Based on the above observation we can construct a zonal transmission access tariff that takes the form of an insurance premium for insuring the option to sell power in the zonal spot market. The access fee collected by the ISO, is set to the actuarial value of the option,  $X(v) = E\{\text{Min}[0, S_1 - v]\}$  which depends on the insurance level that is defined by the strike price  $v$  ( $X(v)$  is a decreasing function of  $v$ ). Purchasing insurance level  $v$  for one unit of injection entitles the trader to either physical access to the local grid or to

compensation in the amount of the revealed opportunity cost, i.e.,  $\text{Max}[0, S_1 - v]$ . The ISO can then manage congestion so as to minimize compensation payments to curtailed transactions, net of the interzonal congestion fee revenues. Thus a higher insurance level (lower  $v$ ) entails a higher priority, which at the same location implies a higher probability of network access. The access probability corresponding to the same insurance level varies, however, across locations and over time depending on the network load conditions.

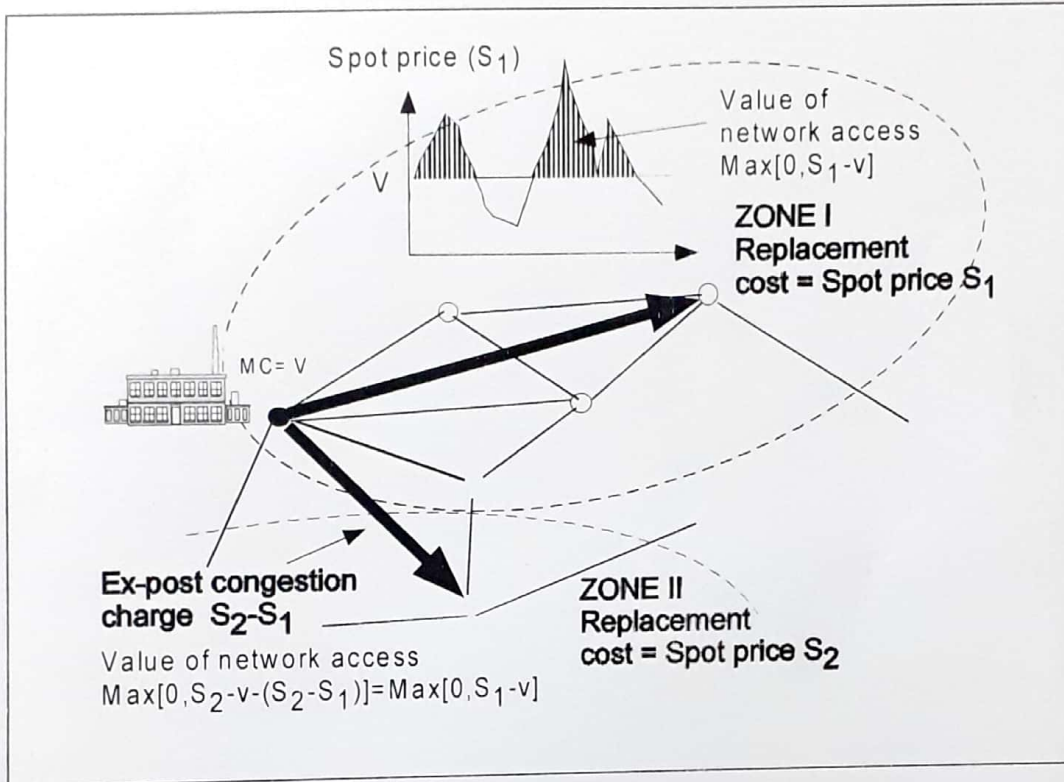


Figure 5

### Valuation of Network Access as An Option

It can be shown that under the above scheme, if every trader insures its transactions revealing the true marginal cost, then minimizing net compensation is equivalent to economic dispatch. However, traders will tend to balance the risk of curtailment with the gain from a lower transmission cost and will have incentives to share the risk of curtailment and underinsure (select a strike price higher than the true marginal cost) in order to lower transmission access payments. This incentive causes distortion of the economic signal. Such distortion is the price we pay for having a uniform tariff across the entire zone (differentiated only according to priority) rather than node specific pricing. Nevertheless, the economic signals are in the right direction and preliminary simulations described by Deng and Oren [1998] suggest that the efficiency losses due to "inaccuracy" of the economic signals are modest.

### Conclusion

Short term theoretical efficiency claims of the nodal pricing approach to transmission tariffs and congestion management are based on unrealistic assumptions and a myopic view of optimal resource use. Hence, such claims do not justify the burden of thousands of different prices, the drawbacks of ex-post transmission charges and the constraints imposed on bilateral transactions and on risk management. Zonal aggregation and decoupling of the energy market from congestion relief protocols simplify transmission



pricing and congestion management without substantial short-term efficiency losses. Such simplifications offer more certainty of transmission cost to traders and better risk management capability. They also enable a higher degree of decentralization in the energy market, which will promote innovation and long-term efficiency.

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